National Curriculum for

MATHEMATICS

Grades I – XII

2006

GOVERNMENT OF PAKISTAN
MINISTRY OF EDUCATION
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National Curriculum for Mathematics 2006 – Grades I-XII
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Contents

Introduction

The Curriculum

The term 'curriculum' etymologically comes from the Latin root which means 'race course' where the words 'race' and 'course' are suggestive of the time and the path respectively. The curriculum, therefore, can be seen as the prescribed course of study to be covered in a specific timeframe. As a field of study, evolution of curriculum began in 1890's and the first book devoted to the theme entitled 'The Curriculum' was published in 1918 by Franklin Bobbitt.

The Importance of Mathematics

An information- and technology-based society requires individuals, who are able to think critically about complex issues, analyze and adapt to new situations, solve problems of various kinds and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills and habits of mind that are essential for successful and rewarding participation in such a society. The more the technology is developed the greater the level of mathematical skill is required.

Mathematical structures, operations and processes provide students with a framework and tools for reasoning, justifying conclusions and expressing ideas clearly. As students identify relationships between mathematical concepts and everyday situations and make connections between Mathematics and other subjects, they develop the ability to use Mathematics to extend and apply their knowledge in other fields.

National Curriculum for Mathematics

The first National Curriculum was developed in 1975-76 which was further reviewed in 1984-85 and then in 1994-95. The next review took place in the years 2000 and 2002 for science and social science subjects respectively.

Since new technological revolution has tremendous impact on society, it is causing and will increasingly cause educational aims to be re-thought, making curriculum development a dynamic process. The present curriculum reform of all subjects for grades I to XII is underway. The main objectives are to make the curriculum more vibrant and more responsive to the modern, socio-economic, technical, professional and labour market needs of the country. It should be improved and uplifted to make it comparable with international standards.

The most important feature of this curriculum is its continued focus on the content of the Mathematics standards. The goal of accelerating the progress of students, through a standard-based programme, has a significant impact on the entire curriculum. These standards necessitate the provision of more continued, more substantive, more rigorous and more systematic instructions to students. In addition, the benchmarks in each of the content areas are highlighted to further elaborate the standards. They provide indicators of expectations from the students at completion of each of the five developmental stages: stage one (grade I to II), stage two (grade III to V), stage three (grade VI to VIII), stage four (grade IX to X) and stage five (grade XI to XII). Learning outcomes indicate what students should know and be able to do for each topic at the particular development level.

The following themes permeate the National Curriculum for Mathematics.

- The curriculum is designed to help students build the solid conceptual foundation in Mathematics that will enable them to apply their knowledge skilfully and further their learning successfully.
- The curriculum emphasizes on the geometrical concepts that enable the students to think logically, reason systematically and conjecture astutely.
- The curriculum stresses graphics that enable the students to visualize and interpret mathematical expressions correctly rather to manipulate them 'blindly'.
- The curriculum recognizes the benefits that current technologies can bring to the learning and doing mathematics. It, therefore, integrates the use of appropriate technologies to enhance learning in an ever increasingly information-rich world.

In the National Curriculum for Mathematics teachers' role has been rerouted that shifts from 'dispensing information' to planning investigative tasks, managing a cooperative learning environment and supporting students' creativity in developing rational understanding of the concepts of Mathematics.

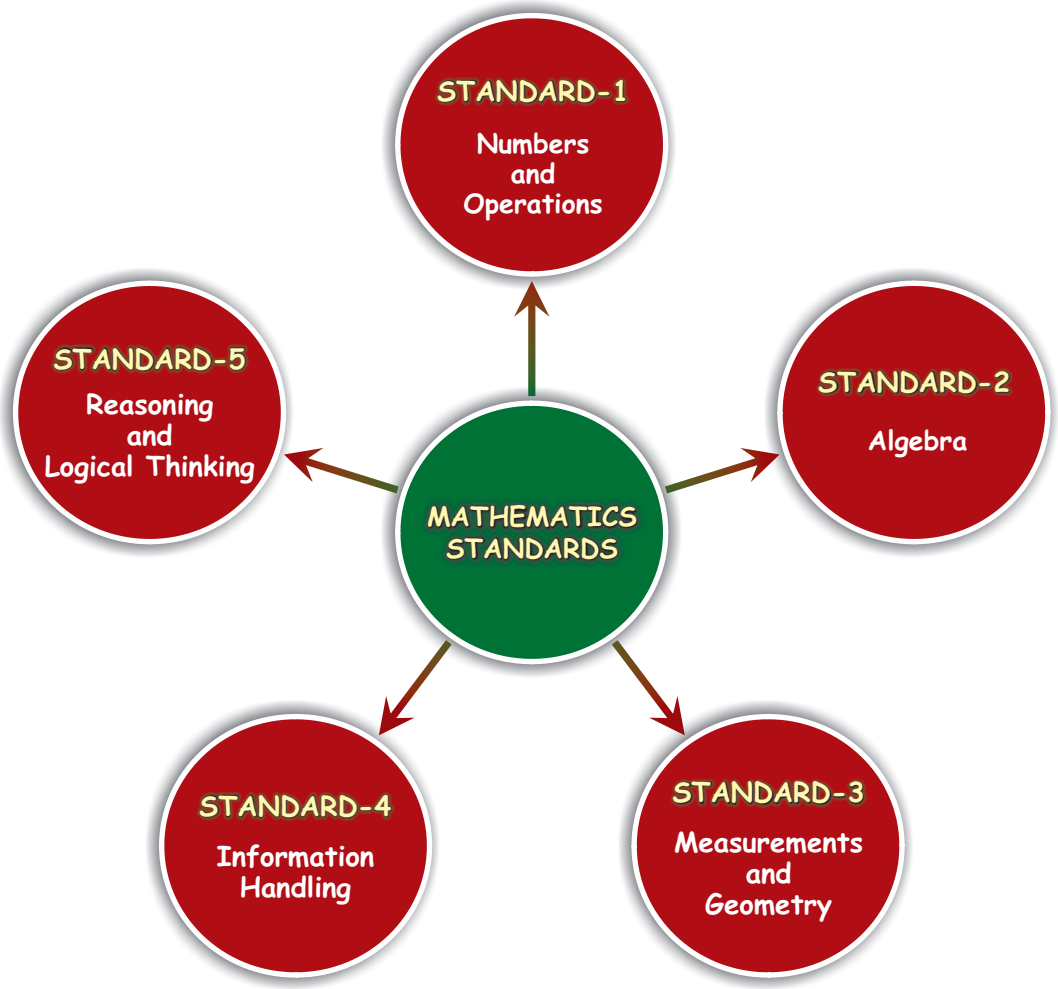
To ensure that assessment and evaluation are based on curriculum expectations and the achievement levels outlined in the curriculum, specific strategies are suggested that lead to the improvement of student learning. An effective learning-outcomes-oriented quality assurance system, that is based on constant monitoring and effective feedback loops, is recommended.

Print materials, particularly the textbooks, have to play a key role towards providing quality education at all levels. Although there are many stakeholders that contribute towards the overall learning of the child yet the importance of textbook as a reservoir of information/knowledge cannot be ignored. In addition to the textbook, teaching and learning resources include teacher's manual, workbook and electronic resources. The guidelines to develop these resources are elaborated.

Standards and Benchmarks

Introduction

National Curriculum for Mathematics is comprised of five standards. The competencies are intentionally kept broad as to allow flexibility to the teachers in accordance with their students.



The benchmarks, thereafter, serve as a guide indicating how competencies at a particular developmental level are to be attained in order to meet the standards. They provide indicators of expectations from students at completion of each of five developmental stages: grades I to II, III to V, VI to VIII, IX to X and XI to XII.

NUMBERS AND OPERATIONS

STANDARD-1	The students will be able to • identify numbers, ways of representing numbers and effects of operations in various situations, • compute fluently with fractions, decimals and percents, • manipulate different types of sequence and apply operations on matrices.

BENCHMARKS

Grades I-II	Grades III-V	Grades VI-VIII	Grades IX-X	Grades XI-XII
• Count, read and write numbers up to 999. • Write numbers up to 100 in words and write ordinal numbers up to 20. • Identify the place value of each digit in a 3-digit number. • Add and subtract up to 3-digit numbers. • Multiply numbers within multiplication tables of 2, 3, 4, 5 and 10. • Divide numbers within multiplication tables of 2, 3, 4, 5 and 10 with remainder zero. • Recognize and represent unit fractions up to $\frac{1}{12}$.	• Read and write Roman numbers up to 20. • Read, write, compare, and identify place values of numbers up to 1 000 000 000. • Add and subtract numbers of complexity and of arbitrary size. • Multiply and divide up to 6-digit numbers by 2- and 3- digit numbers. • Distinguish between even and odd, prime and composite numbers. • Differentiate between factors and multiples. • Calculate HCF (LCM) of three (four) 2-digit numbers using prime factorization and division method. • Use four basic operations on fractions. • Convert percentage to fraction and to decimal and vice versa. • Calculate unit rate, direct and inverse proportions. • Add and subtract measures of distance, time and temperature.	• Identify different types of set with notations. • Verify commutative, associative, distributive and De Morgan's laws w.r.t. union and intersection of sets and illustrate them through Venn diagrams. • Identify and compare integers, rational and irrational numbers. • Apply basic operations on integers and rational numbers and verify commutative, associative and distributive properties. • Arrange absolute values of integers in ascending and descending order. • Find HCF and LCM of two or more numbers using division and prime factorization. • Convert numbers from decimal system to numbers with base 2, 5 and 8 and vice versa. • Add, subtract and multiply numbers with base 2, 5 and 8. • Apply the laws of exponents to evaluate expressions. • Find square and square root, cube and cube root of a real number. • Solve problems on ratio, proportion, profit, loss, mark-up, leasing, zakat, ushr, taxes, insurance and money exchange.	• Add, subtract and multiply matrices. • Evaluate determinant and inverse of a matrix of order 2-by-2. • Explain real numbers with its properties and depict them on the number line. • Distinguish between real and complex numbers and apply four operations on complex numbers. • Apply laws of logarithm. • Identify Cartesian product and binary relation. • Identify function, its domain, co-domain and range. • Prove the fundamental properties of union and intersection of two or three sets. • Apply basic operations on surds of second order. • Calculate ratio, proportions, variations, joint variations. • Apply theorems on proportions.	• Identify complex numbers and their properties and carryout basic operations on complex numbers. • Evaluate determinant, find inverse and rank of a matrix. • Explain and construct various sequences, and series of real numbers. • Apply principle of Mathematical Induction to prove statements, identities and formulae. • Find approximate values of the binomial expansions having indices as rational numbers. • Integrate technology to aid the process of mathematical exploration.

ALGEBRA

STANDARD-2	The students will be able to • analyze number patterns and interpret mathematical situations by manipulating algebraic expressions and relations, • model and solve contextualized problems, • interpret functions, calculate rate of change of functions, integrate analytically and numerically, determine orthogonal trajectories of a family of curves and solve non-linear equations numerically.

BENCHMARKS

Grades I-II	Grades III-V	Grades VI-VIII	Grades IX-X	Grades XI-XII
• Analyze patterns and relationships with respect to size, number, colour/shape and other properties.	• Explain and analyze patterns, identify missing numerals and elements in a pattern or sequence and determine a rule for repeating and extending patterns. • Use symbolic notation to represent a statement of equality.	• Identify algebraic expressions and basic algebraic formulas. • Apply four basic operations on polynomials. • Manipulate algebraic expressions using formulas. • Formulate linear equations in one and two variables. • Solve simultaneous linear equations using different techniques.	• Factorize algebraic expressions. • Apply remainder/ factor theorem to verify that a first degree polynomial is a factor of a given polynomial. • Find HCF and LCM and square root of algebraic expressions. • Solve system of two linear equations in two unknowns by elimination, graphical and matrix methods. • Solve linear inequalities with rational coefficients. • Solve equations (quadratic and reducible to quadratic). • Analyze attributes of quadratic equations. • Form a quadratic equation from the given roots. • Resolve rational expressions into partial fractions.	• Solve equations with complex coefficients. • Solve system of homogeneous and non-homogeneous linear equations by appropriate method (matrix inversion, Gauss elimination, Gauss Jordan and Cramer's rule). • Analyze attributes of functions and give their graphical representation. • Evaluate limits of functions. • Differentiate and integrate algebraic and transcendental functions. • Find extreme values of a function. • Solve ordinary differential equations of first order and first degree. • Find orthogonal trajectories of a given family of curves. • Find partial derivatives of a function of two variables and verify Euler theorem. • Solve non-linear equations in one variable by numerical techniques. • Evaluate definite integrals by numerical methods. • Integrate technology to aid the process of mathematical exploration.

MEASUREMENTS AND GEOMETRY

STANDARD-3	The students will be able to • identify measurable attributes of objects, construct angles and two dimensional figures, • analyze characteristics and properties of geometric shapes and develop arguments about their geometric relationships, • recognize trigonometric identities, analyze conic sections, draw and interpret graphs of functions.

BENCHMARKS

Grades I-II	Grades III-V	Grades VI-VIII	Grades IX-X	Grades XI-XII
• Identify and apply measurable attributes of length, weight/ mass, capacity/ volume and time. • Identify square, rectangle, triangle, circle and oval.	• Add, subtract and convert standard units of length, weight/ mass, capacity/ volume, time and temperature. • Draw, label and classify lines, angles, quadrilaterals and triangles based on their properties. • Determine the perimeter and area of a square, rectangle and triangle using formulas.	• Draw and subdivide a line segment and an angle. • Construct triangle (given SSS, SAS, ASA, RHS), parallelogram and segments of a circle. • Apply properties of lines, angles and triangles to develop arguments about their geometric relationships. • Apply appropriate formulas to calculate perimeter and area of quadrilateral, triangular and circular regions. • Determine surface area and volume of cube, cuboid, sphere, cylinder and cone. • Find trigonometric ratios of acute angles and use them to solve right angled triangles.	• Use distance formula to find distance between two points in Cartesian plane. • Convert degrees into radians and vice versa. • Calculate length of an arc and area of a sector of a circular region. • Use trigonometric identities to verify relationships between trigonometric ratios. • Apply characteristics and properties of angles, triangles, parallelograms and circles to develop arguments about their geometric relationships. • Draw external (internal) tangent to two equal/unequal circles. • Circumscribe/inscribe /escribe a circle to a given triangle. • Circumscribe/inscribe a square and an equilateral triangle in a given circle.	• Identify vectors in space and apply vector addition, dot/ cross product, scalar triple product. • Differentiate and integrate vector functions. • Use appropriate laws of trigonometry to solve the triangles. • Apply trigonometric formulas to find area of a triangle. • Find radii of circles connected with triangles and prove their relationships. • Interpret graphically the algebraic and transcendental functions. • Interpret and recognize equations of a straight line in standard form. • Show the concurrency of right bisectors/ medians/altitudes of a triangle. • Identify and analyze conic sections (circle, parabola, ellipse and hyperbola). • Integrate technology to aid the process of mathematical exploration.

INFORMATION HANDLING

STANDARD-4	The students will be able to collect, organize, analyze, display and interpret data/ information.

BENCHMARKS

Grades I-II	Grades III-V	Grades VI-VIII	Grades IX-X	Grades XI-XII
	• Compare data and interpret quantities represented on charts, tables and different types of graphs (pictogram and bar) and make predictions based on the information.	• Read, display and interpret bar and pie graphs. • Collect and organize data, construct frequency tables and histograms to display data. • Find measure of central tendency (mean, median and mode).	• Collect data from variety of sources and construct frequency table (distribution) with equal and unequal class intervals. • Find measures of central tendency and dispersion to draw conclusions. • Draw, interpret and identify the graph of a linear function.	• Solve real life problems involving arithmetic/geometric sequence and series. • Use partial fractions to find sum to n terms and to infinity the series. • Find sum to n terms of arithmetico-geometric series. • Solve problems involving permutation and combination. • Use various methods to solve probability problems. • Integrate technology to aid the process of mathematical exploration.

REASONING AND LOGICAL THINKING

STANDARD-5	The students will be able to • use patterns, known facts, properties and relationships to analyze mathematical situations, • examine real life situations by identifying, mathematically valid arguments and drawing conclusion to enhance their mathematical thinking.

BENCHMARKS

Grades I-II	Grades III-V	Grades VI-VIII	Grades IX-X	Grades XI-XII
• Sort, classify and compare familiar shapes. • Apply analytical reasoning to explain features of a shape.	• Communicate reasoning about patterns and geometric figures. • Explain method and reasoning when solving problems involving numbers and data.	• Find different ways of approaching a problem to develop logical thinking and explain their reasoning. • Solve problems using mathematical relationships and present results in an organized way. • Construct and communicate convincing arguments for geometric situations.	• Investigate general statements. • Choose appropriate strategy to solve mathematical problems. • Formulate and test logical arguments about geometric figures and patterns and communicate reasoning. • Show step by step deduction in solving a problem, explain and justify how they arrived at a conclusion.	• Identify valid and invalid arguments. • Apply mathematical ideas and arguments logically. • Use graphics to optimize a situation. • Acquire ability to apply mathematics in physical situations and use numerical techniques to find approximate solution. • Develop and communicate logical proofs and counter examples for geometrical and mathematical statements.

CURRICULUM FOR MATHEMATICS – GRADE I

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 CONCEPT OF WHOLE NUMBERS

1.1 Numbers 0 – 9		
1.1.1 Numbers 1 – 9	i)	Identify numbers 1 – 9.
1.1.2 Concept of 0	ii)	Identify 0 as a number.
1.1.3 Numbers 0 – 9	iii)	Read numbers up to 9 in numerals and in words.
	iv)	Write numbers up to 9 in numerals and in words.
	v)	Count objects up to 9 and represent in numbers.
	vi)	Match the numbers 0 – 9 with objects.
	vii)	Count backward from 9.
	viii)	Arrange numbers in ascending and descending order.
	ix)	Identify which number (up to 9) comes • before/after a number, • between two numbers.
1.2 Numbers up to 100		
1.2.1 Concept of 10	i)	Identify 10 as a number.
	ii)	Compare and order the numbers 0 – 10.
1.2.2 Numbers up to 99	iii)	Read numbers up to 99.
	iv)	Write numbers up to 99.
	v)	Count numbers up to 99.
1.2.3 Concept of Place Values	vi)	Recognize the place values of numbers (tens and ones).
	vii)	Identify the place value of the specific digit in a two digit number.
	viii)	Compare one and two digit numbers.
	ix)	Write numbers in increasing and decreasing order.

1.2.4 Concept of 100 1.2.5 Cardinal and Ordinal Numbers	x) xi) xii) xiii) xiv) xv) xvi) xvii)	Place the mixed numbers in order. Order the set of numbers from 0 to 99 in increasing and decreasing order. Identify which number (up to 99) comes • before/after a number, • between two numbers. Write numbers in increasing and decreasing order up to 99. Count in tens and recognize 100 as a number. Identify and write missing numbers in a sequence from 1 to 100. Count and write the number of objects in a given set. Identify the position of objects using ordinal numbers such as first, second, …, tenth, including representations 1^{st}, 2^{nd} etc.
1.3 Comparing and Ordering	i) ii) iii)	Compare two or more groups in terms of number of objects. Match objects having one to one correspondence. Identify the number of objects in two groups to show 'more than' or 'less than'.
GUIDANCE FOR AUTHOR • Exclude use of symbols '>' and '<'. • In comparison of numbers up to 100 use the terms 'more than' and 'less than'. • Exclude using the term ordinal number.		

UNIT 2 NUMBER OPERATIONS

2.1 Addition 2.1.1 Addition and Equality Symbols	i) ii)	Compare numbers from 1 to 20 to identify 'How much more' one is from the other. Recognize and use symbols of addition '+' and equality '='.

2.1.2 Addition of Numbers (without carrying)	iii)	Add two one-digit numbers (sum up to 9).
	iv)	Add a two-digit number with one-digit number.
	v)	Add a two-digit number with 10s.
	vi)	Add two two-digit numbers.
	vii)	Complete equation such as □+ 4 =7 (include questions that sum up to 20).
	viii)	Add mentally the numbers using real life examples.
	ix)	Construct addition equations from given pictures.
2.2 Subtraction	i)	Compare numbers from 20 and find 'How much smaller?'
2.2.1 Subtraction Symbol	ii)	Recognize and use the symbol of subtraction '—'.
2.2.2 Subtraction of Numbers (without borrowing)	iii)	Subtract ones from ones.
	iv)	Subtract ones from 2-digit numbers.
	v)	Subtract tens from 2-digit numbers.
	vi)	Subtract 2-digit numbers from 2-digit numbers.
	vii)	Fill up the equation, such as 9 — □ = 7, with proper number.
	viii)	Subtract mentally the numbers given in simple real life examples.
	ix)	Construct subtraction equation from given pictures.

UNIT 3 MEASUREMENT OF LENGTH AND MASS

3.1 Comparison of Objects	Compare objects to identify: • long, longer, longest, • short, shorter, shortest, • tall, taller, tallest, • high, higher, highest, • heavy, heavier, heaviest, • light, lighter, lightest.

GUIDANCE FOR AUTHOR • Exclude finding the amount of difference in length/ weight.

UNIT 4 MONEY

4.1 Pakistani Currency	i) ii)	Identify Pakistani currency coins (Rs. 1, 2 and 5). Identify Pakistani currency notes (Rs. 10, 20, 50 and 100).
4.2 Equivalent Sets of Money	i) ii)	Match a group of coins/notes to an equivalent group of different denominations. Add and subtract money using the prices of objects (e.g. toys).
4.3 Comparing Money	i) ii) iii)	Recognize money change (up to 100) to its equivalent denominations. Determine if enough money is available to make a purchase. Add different combinations of coins/notes.

UNIT 5 TIME AND DATE

5.1 Time	i) ii) iii)	Recognize the hour and minute hands of an analog clock. Read and tell time in hours from the analog clock e.g., two o'clock. Read and tell time in hours from the digital clock.
5.2 Date	i) ii) iii)	Name in order the days of the week. Identify which day comes after/before a particular day. Name (orally) the solar months of the year.
GUIDANCE FOR AUTHOR • Exclude use of 24-hour clock.		

UNIT 6 GEOMETRY

6.1 Identification of Basic Shapes	i) Recognize and match objects, from daily life, of similar shape. ii) Identify the following basic shapes: • rectangle, • square, • circle, • oval, • triangle. iii) Identify the basic shapes from real life objects. iv) Match similar basic shapes.
6.2 Patterns	i) Identify and describe patterns with 2 or 3 elements. ii) Extend a given pattern of 2 to 3 elements.
6.3 Position	Identify whether an object is placed • inside or outside, • above or below, • over or under, • far or near, • before or after, • right or left, of a given picture.

CURRICULUM FOR MATHEMATICS – GRADE II

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 NUMBERS

1.1 Numbers		
1.1.1 Ordinal Numbers	i)	Write ordinal numbers from first to twentieth.
1.1.2 Numbers in Words	ii)	Write numbers 1 – 100 in words.
1.2 Numbers up to 1000		
1.2.1 Place Value	i)	Recognize the place value of a 3-digit number.
	ii)	Identify the place value of a specific digit in a 3-digit number.
	iii)	Compare 2- or 3-digit numbers (hundreds, tens and ones).
	iv)	Read numbers up to 999.
	v)	Write numbers up to 999 in numerals.
	vi)	Identify numbers given in ascending or descending order.
	vii)	Count backward ten step down from any given number.
	viii)	Arrange numbers up to 999, written in mixed form, in increasing or decreasing order.
1.2.2 Counting in Tens and Hundreds	ix)	Count and write in 10s (e.g. 10, 20, 30, …).
	x)	Count and write in 100s (e.g. 100, 200, 300, …).
	xi)	Identify the smallest/largest number in a given set of numbers.
	xii)	Recognize that 1000 is one more than 999 and the first four digit number.
1.3 Fractions	i)	Recognize fraction as equal parts of a whole.

	ii)	Identify half, one third and quarter with the help of objects and figures (without writing $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$).
	iii)	Represent half, one third and quarter in numerical form as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.
	iv)	Shade the equal parts of a given figure to match a given fraction.
	v)	Recognize and name unit fractions up to $\frac{1}{12}$.
	vi)	Recognize fractions like two third, three fourth, four fifth and so on using $\frac{2}{3}, \frac{3}{4}, \cdots$.

UNIT 2 NUMBER OPERATIONS

2.1 Addition		
2.1.1 Addition of 2-digit Numbers (with carrying)	i)	Add ones and ones.
	ii)	Add ones and 2-digit numbers with carrying.
	iii)	Add 2-digit numbers and 2-digit numbers with carrying.
	iv)	Solve real life problems, involving addition of 2-digit numbers, with carrying.
2.1.2 Addition of 3-digit Numbers (without carrying)	v)	Add 3-digit numbers and ones without carrying.
	vi)	Add 3-digit numbers and 2-digit numbers without carrying.
	vii)	Add 3-digit numbers and 3-digit numbers without carrying.
	viii)	Solve real life problems, involving addition of 3-digit numbers, without carrying.
2.1.3 Addition of 3-digit Numbers (with carrying)	ix)	Add 3-digit numbers and ones with carrying of tens and hundreds.
	x)	Add 3-digit numbers and 2-digit numbers with carrying of tens and hundreds.

	xi)	Add 3-digit numbers and 3-digit numbers with carrying of tens and hundreds.
	xii)	Solve real life problems with carrying of tens and hundreds.
2.1.4 Commutative Property	xiii)	Verify commutative property with respect to addition (sum should not exceed 100).
2.2 Subtraction		
2.2.1 Subtraction of 2-digit Numbers (with borrowing)	i)	Subtract ones from 2-digit numbers with borrowing.
	ii)	Subtract 2-digit numbers from 2-digit numbers with borrowing.
	iii)	Solve real life problems of subtraction with borrowing.
2.2.2 Subtraction of 3-digit Numbers (without borrowing)	iv)	Subtract ones from 3-digit numbers without borrowing.
	v)	Subtract 2-digit numbers from 3-digit numbers without borrowing.
	vi)	Subtract 3-digit numbers from 3-digit numbers without borrowing.
	vii)	Solve real life problems of subtraction without borrowing.
2.2.3 Subtraction of 3-digit Numbers (with borrowing).	viii)	Subtract ones from 3-digit numbers with borrowing.
	ix)	Subtract 2-digit numbers from 3-digit numbers with borrowing.
	x)	Subtract 3-digit numbers from 3-digit numbers with borrowing.
	xi)	Solve real life problems of subtraction with borrowing.
2.2.4 Addition and Subtraction	xii)	Solve simple problems regarding addition and subtraction with carrying/borrowing in mixed form.
2.3 Multiplication	i)	Recognize and use multiplication symbol '×'.

	ii)	Recognize multiplication as repeated addition (e.g. $2+2+2=6 \Leftrightarrow 3 \text{ times } 2=3\times 2=6$).
	iii)	Complete number sequences in steps of 2, 3, 4, 5 and 10 (e.g. in steps of 2 the sequence is expressed as 2, 4, 6, …).
	iv)	Develop multiplication tables of 2, 3, 4, 5 and 10 till the multiplication 10×10.
	v)	Multiply numbers within multiplication table.
	vi)	Verify commutative property of multiplication.
	vii)	Solve real life problems on multiplication.
2.4 Division	i)	Recognize and use division symbol '÷'.
	ii)	Recognize division as successive subtraction.
	iii)	Divide numbers within the multiplication tables with remainder zero.
	iv)	Solve real life problems involving division.
2.5 Addition, Subtraction, Multiplication and Division		Solve real life problems (using Pakistani currency as well) involving addition, subtraction, multiplication and division.

UNIT 3 MEASUREMENT OF LENGTH, MASS AND CAPACITY

3.1 Measurement of Length	i)	Recognize the standard units of length, i.e. metre, centimetre.
	ii)	Read and write standard units of length including abbreviations.
	iii)	Use appropriate units of length to measure (with straightedge/ ruler) the objects.
	iv)	Solve real life problems involving measurements.
3.2 Measurement of Mass/ Weight	i)	Recognize the standard units of mass/ weight, i.e. kilogram, gram.

	ii)	Read and write standard units of mass/ weight including abbreviations.
	iii)	Solve real life problems involving mass/ weight.
3.3 Measurement of Capacity	i)	Compare capacity of different objects (jug, glass, cup etc.).
	ii)	Recognize and use the standard unit of capacity/ volume, i.e. litre.
	iii)	Read and write standard units of capacity/ volume including abbreviations.
	iv)	Solve real life problems involving capacity/ volume.

UNIT 4 TIME

4.1 Time	i)	Know the number of hours in a day and number of minutes in an hour.
	ii)	Read and write the time from a clock in hours and minutes (with five minute intervals) e.g., read 8:15 as eight fifteen and 8:50 as eight fifty.
	iii)	Recognize a.m. and p.m.
	iv)	Draw hands of a clock to show time in hours and minutes (with five minute intervals).
	v)	Use solar calendar to find a particular date.
	vi)	Use lunar calendar to find a particular date.

UNIT 5 GEOMETRY

5.1 Two-dimensional Figures	i)	Identify the figures like square, rectangle, triangle, circle, semi-circle and quarter-circle.
	ii)	Identify vertices and sides of a triangle, rectangle and square.
5.2 Lines and Curves	i)	Differentiate between a straight line and a curved line.

	ii)	Identify straight and curved lines from the given line drawings.
	iii)	Use straightedge/ ruler to draw a straight line of given length (exclude fractional lengths).

CURRICULUM FOR MATHEMATICS – GRADE III

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 NUMBERS

1.1 Roman Numbers	i) ii)	Read Roman numbers up to 20. Write Roman numbers up to 20.
1.2 Even and Odd Numbers	i) ii)	Identify even and odd numbers up to 99 within a given sequence. Write even or odd numbers within a given sequence.
1.3 Place Values		Identify the place values of numbers up to 6-digits.
1.4 Numbers up to 100,000		Read and write given numbers up to 100,000 (hundred thousand) in numerals and in words.
1.5 Comparing and Ordering the Numbers	i) ii)	Compare two numbers using symbols '<', '>' and '='. Write the given set of numbers in ascending and descending order.
1.6 Number Line	i) ii)	Represent a given number on number line. Identify the value of a number from number line.

UNIT 2 NUMBER OPERATIONS

2.1 Addition	i) ii) iii)	Add numbers up to four digits (with and without carrying) vertically and horizontally. Add numbers up to 100 using mental calculation strategies. Solve real life problems involving addition.
2.2 Subtraction	i)	Subtract numbers up to four digits with and without borrowing.

	ii)	Subtract numbers up to 100 using mental calculation strategies.
	iii)	Solve real life problems involving subtraction.
2.3 Multiplication	i)	Use the term 'product' for multiplication of two numbers.
	ii)	Develop multiplication tables for 6, 7, 8 and 9.
	iii)	Multiply 2-digit numbers by 1-digit numbers.
	iv)	Multiply a number by zero.
	v)	Apply mental mathematical strategies to multiply numbers up to the table of 10.
	vi)	Solve real life problems involving multiplication of 2-digit numbers by 1-digit numbers.
2.4 Division	i)	Divide 2-digit numbers by 1-digit numbers (with zero remainder).
	ii)	Apply mental mathematical strategies to divide numbers up to the table of 10.
	iii)	Solve real life problems involving division of 2-digit numbers by 1-digit numbers.

UNIT 3 FRACTIONS

3.1 Common Fractions	i)	Express the fractions in figures and vice versa.
	ii)	Match the fractions with related figures.
3.2 Equivalent Fractions	i)	Identify equivalent fractions from the given figures.
	ii)	Write three equivalent fractions for a given fraction.
3.3 Proper and Improper Fractions		Differentiate between proper and improper fraction.
3.4 Comparing Fractions		Compare fractions, with same denominators, using symbols '<', '>' and '='.

3.5 Addition of Fractions	i) ii)	Add two fractions with same denominators. Represent addition of fractions through figures.
3.6 Subtraction of Fractions	i) ii)	Subtract fractions with same denominators. Represent subtraction of fractions through figures.

UNIT 4 MEASUREMENT OF LENGTH, MASS AND CAPACITY

4.1 Length		
4.1.1 Units of Length	i)	Read standard units of length (kilometre, metre and centimetre) including abbreviations.
	ii)	Measure and write standard units of length including abbreviations.
4.1.2 Addition of Units of Length	iii)	Add measures of length in same units with and without carrying.
	iv)	Solve real life problems involving same units of length for addition with and without carrying.
4.1.3 Subtraction of Units of Length	v)	Subtract measures of length in same units with and without borrowing.
	vi)	Solve real life problems involving same units of length for subtraction with and without borrowing.
4.2 Mass/ Weight		
4.2.1 Units of Mass/ Weight	i)	Read standard units of mass/ weight (kilogram and gram) including abbreviations.
	ii)	Measure and write standard units of mass/ weight including abbreviations.
4.2.2 Addition of Units of Mass/ Weight	iii)	Add measures of mass/ weight in same units with and without carrying.
	iv)	Solve real life problems involving same units of mass/ weight for addition with and without carrying.
4.2.3 Subtraction of Units of	v)	Subtract measures of mass/ weight in same units

Mass/ Weight		with and without borrowing.
	vi)	Solve real life problems involving same units of mass/ weight for subtraction with and without borrowing.
4.3 Volume/Capacity		
4.3.1 Units of Volume	i)	Read standard units of volume (litre and millilitre) including abbreviations.
	ii)	Measure and write standard units of volume including abbreviations.
4.3.2 Addition of Units of Volume	iii)	Add measures of volume in same units with and without carrying.
	iv)	Solve real life problems involving same units of volume for addition with and without carrying.
4.3.3 Subtraction of Units of Volume	v)	Subtract measures of volume in same units with and without borrowing.
	vi)	Solve real life problems involving same units of volume for subtraction with and without borrowing.

UNIT 5 TIME

5.1 Units of Time	i)	Use a.m. and p.m. to record the time from 12-hour clock.
	ii)	Read and write time from analog and digital clocks.
	iii)	Read and write days and dates from the calendar.
5.2 Addition of Units of Time	i)	Add units of time in hours.
	ii)	Solve real life problems involving units of time for addition in hours.
5.3 Subtraction of Units of Time	i)	Subtract units of time in hours.
	ii)	Solve real life problems involving subtraction of units of time in hours.

UNIT 6 GEOMETRY

6.1 Geometrical Shapes	i) ii) iii)	Recognize point, line segment, ray. Classify figures according to number of sides as quadrilaterals (rectangles, squares) and triangles. Identify circle, its radius and diameter.
6.2 Perimeters		Calculate perimeters of squares, rectangles and triangles.

UNIT 7 DATA REPRESENTATION

7.1 Picture Graphs	Read and interpret a picture graph.

CURRICULUM FOR MATHEMATICS – GRADE IV

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 NUMBERS AND ARITHMETIC OPERATIONS

1.1 Numbers	i) Identify place values of digits up to one hundred million. ii) Read numbers up to one hundred million. iii) Write numbers up to one hundred million. iv) Recognize numbers in words up to one hundred million. v) Compare and order numbers up to 8 digits.
1.2 Addition	i) Add numbers up to 6 digits. ii) Solve real life problems involving addition of numbers up to 6 digits.
1.3 Subtraction	i) Subtract numbers up to 6 digits. ii) Solve real life problems involving subtraction of numbers up to 6 digits.
1.4 Multiplication	i) Multiply numbers up to 5 digits by numbers up to 3 digits. ii) Solve real life problems involving multiplication.
1.5 Division	i) Divide numbers up to 4 digits by numbers up to 2 digits. ii) Solve real life problems involving division
1.6 Addition, Subtraction, Multiplication and Division	i) Use mixed operations of addition & subtraction and multiplication & division ii) Solve real life problems (using Pakistani currency as well) involving addition, subtraction, multiplication and division.

UNIT 2 FACTORS AND MULTIPLES

2.1 Divisibility Tests	i)	Identify divisibility rules for 2, 3, 5 and 10.
	ii)	Use divisibility tests for 2, 3, 5 and 10 on numbers up to 5 digits.
2.2 Prime and Composite Numbers	i)	Define prime and composite numbers.
	ii)	Differentiate between prime and composite numbers.
2.3 Factors and Multiples	i)	List factors of a number up to 50.
	ii)	List the first twelve multiples of a 1-digit number.
	iii)	Differentiate between factors and multiples.
2.4 Prime Factorization		Factorize a number by using prime factors.
2.5 Highest Common Factor (HCF)	i)	Determine common factors of two or more 2-digit numbers.
	ii)	Find HCF of two or more 2-digit numbers using • Venn diagram, • prime factorization.
	iii)	Solve real life problems involving HCF.
2.6 Least Common Multiple (LCM)	i)	Determine common multiples of two or more 2-digit numbers.
	ii)	Find LCM by • common multiples, • prime factorization.
	iii)	Solve real life problems involving LCM.

UNIT 3 FRACTIONS

3.1 Fractions	i)	Define a fraction.
	ii)	Recognize like and unlike fractions.
	iii)	Compare two unlike fractions by converting them to equivalent fractions with the same denominator.

	iv)	Arrange fractions in ascending and descending order.
	v)	Simplify fractions to the lowest form.
3.2 Types of Fraction	i)	Identify unit, proper, improper and mixed fractions.
	ii)	Convert improper fraction to mixed fraction and vice versa.
3.3 Addition and Subtraction of Fractions	i)	Add fractions with unlike denominators.
	ii)	Verify the commutative property of addition of fractions with same denominators.
	iii)	Verify the associative property of addition of fractions with same denominators.
	iv)	Subtract fractions with unlike denominators.
3.4 Multiplication of Fractions	i)	Multiply fractions with whole numbers.
	ii)	Multiply two or more fractions (proper, improper and mixed fractions).
	iii)	Verify the commutative property of multiplication of fractions.
	iv)	Verify the associative property of multiplication of fractions.
3.5 Division of Fractions	i)	Divide a fraction by a whole number.
	ii)	Divide a whole number by a fraction.
	iii)	Divide a fraction by another fraction (proper, improper and mixed fractions).
		Solve real life problems involving fractions using all four operations.

UNIT 4 DECIMALS AND FRACTIONS

4.1 Decimals	i)	Know a decimal number as an alternate way of writing a fraction.
	ii)	Define decimal as a fraction whose denominator is 10 or a power of 10.
	iii)	Recognize the places occupied by the digits, after the decimal point, as decimal places.
	iv)	Identify the place value of a digit in decimals.
4.2 Conversion between Fractions and Decimals	i)	Convert a given fraction to a decimal if • denominator of the fraction is 10 or a power of 10. • denominator of the fraction is not a power of 10 but can be converted to.
	ii)	Convert decimals (up to three decimal places) to fractions.
4.3 Basic Operations on Decimals	i)	Add and subtract decimals (up to two decimal places).
	ii)	Multiply a decimal by 10, 100 and 1000.
	iii)	Multiply a decimal by a 2-digit number.
	iv)	Divide a decimal by a 1-digit number (quotient being a decimal up to two decimal places).
		Solve real life problems involving decimals up to two decimal places.

UNIT 5 MEASUREMENTS

5.1 Length		
5.1.1 Conversion of Units of Length	i)	Convert • kilometres to metres, • metres to centimeters, • centimetres to millimeters.
5.1.2 Addition and	ii)	Add and subtract expressions involving similar units

Subtraction of Units of Length		of length. iii) Use appropriate units to measure the length of different objects. iv) Solve real life problems involving conversion, addition and subtraction of units of length.
5.2 Mass/ Weight 5.2.1 Conversion of Units of Mass/Weight 5.2.2 Addition and Subtraction of Units of Mass/Weight	i) ii) iii) iv)	Convert kilograms to grams. Add and subtract expressions involving similar units of mass/ weight. Use appropriate units to measure the mass/ weight of different objects. Solve real life problems involving conversion, addition and subtraction of units of mass/ weight.
5.3 Volume/Capacity 5.3.1 Conversion of Units of Capacity 5.3.2 Addition and Subtraction of Units of Capacity	i) ii) iii) iv)	Convert litres to millilitres. Add and subtract expressions involving units of capacity/ volume. Use appropriate units to measure the capacity/ volume of different objects (utensils etc). Solve real life problems involving conversion, addition and subtraction of units of capacity/ volume.
5.4 Time 5.4.1 Conversion of Units of Time 5.4.2 Addition and Subtraction Units of Time	i) ii) iii) iv)	Read time in hours, minutes and seconds. Convert hours to minutes and minutes to seconds. Convert years to months, months to days and weeks to days. Add and subtract units of time without carrying /borrowing.

	v)	Solve simple real life problems involving conversion, addition and subtraction of units of time.

UNIT 6 GEOMETRY

6.1 Geometry Box	i)	Know instruments of a Geometry Box i.e., pencil, straightedge/ruler, compasses (sometimes called a pair of compasses), dividers (sometimes called a pair of dividers), set squares and protractor.
	ii)	Recognize the use of pencils of grade H and HB.
	iii)	Demonstrate the use of H and HB pencils by drawing different lines.
6.2 Line	i)	Measure the length of a line in centimetres and millimetres using straightedge/ruler and dividers.
	ii)	Draw a straight line of given length using a straightedge/ruler and dividers.
	iii)	Draw a curved line and measure its length using thread/dividers and straightedge/ruler.
	iv)	Recognize horizontal and vertical lines.
	v)	Draw a vertical line on a given horizontal line using set squares.
	vi)	Recognize parallel and non-parallel lines.
	vii)	Identify parallel and non-parallel lines from a given set of lines.
	viii)	Draw a parallel line to a given straight line using set squares.
	ix)	Draw a line which passes through a given point and is parallel to a given line (using set squares).
6.3 Angle	i)	Recognize an angle through non-parallel lines.
	ii)	Draw an angle AOB with vertex (O) and arms (OA,

		OB) to recognize the notation $\angle$AOB for an angle AOB.
	iii)	Recognize right angle through horizontal and vertical lines.
	iv)	Demonstrate acute and obtuse angles via the right angle.
	v)	Recognize the standard unit for measuring angles as one degree (1°) which is defined as $\frac{1}{360}$ of a complete revolution.
	vi)	Measure angles using protractor where • upper scale of protractor reads the measure of angle from left to right. • lower scale of protractor reads the measure of angle from right to left.
	vii)	Draw a right angle using protractor.
	viii)	Draw acute and obtuse angles of different measures using protractor.
	ix)	Draw an angle (using protractor) • equal in measure of a given angle, • twice the measure of a given angle, • equal in measure of the sum of two given angles.
6.4 Circle	i)	Identify centre, radius, diameter and circumference of a circle.
	ii)	Draw a circle of given radius using compasses and straightedge/ruler.
6.5 Quadrilaterals		Construct squares and rectangles with sides of given measure using protractor, set squares and straightedge/ ruler.

UNIT 7 INFORMATION HANDLING

7.1 Bar Graph	Read and interpret simple bar graphs given in horizontal and vertical form.
7.2 Line graph	Read and interpret line graph.

CURRICULUM FOR MATHEMATICS – GRADE V

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 NUMBERS AND ARITHMETIC OPERATIONS

1.1 Numbers up to one billion	i)	Read numbers up to 1 000 000 000 (one billion) in numerals and in words.
	ii)	Write numbers up to 1 000 000 000 (one billion) in numerals and in words.
1.2 Addition and Subtraction	i)	Add numbers of complexity and of arbitrary size.
	ii)	Subtract numbers of complexity and of arbitrary size.
1.3 Multiplication and Division	i)	Multiply numbers, up to 6 digits, by 10, 100 and 1000.
	ii)	Multiply numbers, up to 6 digits, by a 2-digit and 3-digit number.
	iii)	Divide numbers, up to 6 digits, by a 2-digit and 3-digit number.
		Solve real life problems involving mixed operations of addition, subtraction, multiplication and division.
1.4 Order of Operations: BODMAS Rule	i)	Recognize BODMAS rule, using only parentheses ( ).
	ii)	Carryout combined operations using BODMAS rule.
	iii)	Verify distributive laws.

UNIT 2 HCF AND LCM

2.1 HCF	i)	Find HCF of three numbers, up to 2 digits, using • prime factorization method, • division method.
2.2 LCM	i)	Find LCM of four numbers, up to 2 digits, using

	• prime factorization method, • division method.
	Solve real life problems involving HCF and LCM.

UNIT 3 FRACTIONS

3.1 Addition and Subtraction	Add and subtract two and more fractions with different denominators.
3.2 Multiplication	i) Multiply a fraction by a number and demonstrate with the help of diagrams. ii) Multiply a fraction by another fraction. iii) Multiply two or more fractions involving brackets (proper, improper and mixed fractions). iv) Verify distributive laws. v) Solve real life problems involving multiplication of fractions.
3.3 Division	i) Divide a fraction by a number. ii) Divide a fraction by another fraction (proper, improper and mixed). iii) Solve real life problems involving division of fractions.
3.4 Use of BODMAS Rule	Simplify expressions involving fractions using BODMAS rule.

UNIT 4 DECIMALS AND PERCENTAGES

4.1 Decimals	i) Add and subtract decimals. ii) Recognize like and unlike decimals. iii) Multiply decimals by 10, 100 and 1000. iv) Divide decimals by 10, 100 and 1000.

	v) Multiply a decimal with a whole number. vi) Divide a decimal with a whole number. vii) Multiply a decimal by tenths and hundredths only. viii) Multiply a decimal by a decimal (with three decimal places). ix) Multiply a decimal by a decimal (in the same way as for whole numbers and then put in the decimal point accordingly). x) Divide a decimal by a decimal (by converting decimals to fractions). xi) Divide a decimal by a decimal using direct division by moving decimal positions. xii) Use division to change fractions into decimals. xiii) Simplify decimal expressions involving brackets (applying one or more basic operations). xiv) Round off decimals up to specified number of decimal places. xv) Convert fractions to decimals and vice versa. xvi) Solve real life problems involving decimals.
4.2 Percentages	i) Recognize percentage as a special kind of fraction. ii) Convert percentage to fraction and to decimal and vice versa. iii) Solve real life problems involving percentages.

UNIT 5 DISTANCE, TIME AND TEMPERATURE

5.1 Distance	i) Convert measures given in • kilometers to meters, • meters to centimeters, • centimeters to millimeters, and vice versa.

	ii)	Add and subtract measures of distance.
	iii)	Solve real life problems involving conversion, addition and subtraction of units of distance.
5.2 Time	i)	Convert hours to minutes, minutes to seconds and vice versa.
	ii)	Add and subtract units of time with carrying /borrowing.
	iii)	Convert years to months, months to days, weeks to days and vice versa.
	iv)	Solve real life problems involving conversion, addition and subtraction of units of time.
5.3 Temperature	i)	Recognize units of temperature in Fahrenheit and Celsius.
	ii)	Solve real life problems involving conversion, addition and subtraction of units of temperature.

UNIT 6 UNITARY METHOD

6.1 Unitary Method	i)	Describe the concept of unitary method.
	ii)	Calculate the value of many objects of the same kind when the value of one of these objects is given.
	iii)	Calculate the value of a number of same type of objects when the value of another of the same type is given (unitary method).
6.2 Direct and Inverse Proportion	i)	Define ratio of two numbers.
	ii)	Define and identify direct and inverse proportion.
	iii)	Solve real life problems involving direct and inverse proportion (by unitary method).

UNIT 7 GEOMETRY

7.1 Angles	i)	Recall an angle and recognize acute, right, obtuse, straight and reflex angle.
	ii)	Use protractor to construct • a right angle, • a straight angle, • reflex angles of different measure.
	iii)	Describe adjacent, complementary and supplementary angles.
7.2 Triangles	i)	Define a triangle.
	ii)	Define triangles with respect to their sides (i.e., equilateral, isosceles and scalene triangle).
	iii)	Define triangles with respect to their angles (i.e., acute angled, obtuse angled and right angled triangle).
	iv)	Use compasses and straightedge/ruler to construct equilateral, isosceles and scalene triangles when three sides are given.
	v)	Use protractor and straightedge/ruler to construct equilateral, isosceles and scalene triangles when two angles and included side are given. Measure the lengths of the remaining two sides and one angle of the triangle.
	vi)	Define hypotenuse of a right angled triangle.
	vii)	Use protractor and straightedge/ruler to construct a triangle when two angles and included side are given.
	viii)	Use protractor and straightedge/ruler to construct acute angled, obtuse angled and right angled triangles when one angle and adjacent sides are given.

7.3 Quadrilaterals	i)	Recognize the kinds of quadrilateral (square, rectangle, parallelogram, rhombus, trapezium and kite).
	ii)	Use protractor, set squares and straightedge/ruler to construct square and rectangle with given side(s).

UNIT 8 PERIMETER AND AREA

8.1 Perimeter and Area	i)	Recognize region of a closed figure.
	ii)	Differentiate between perimeter and area of a region.
	iii)	Identify the units for measurement of perimeter and area.
	iv)	Write the formulas for perimeter and area of a square and rectangle.
	v)	Apply formulas to find perimeter and area of a square and rectangular region.
	vi)	Solve appropriate problems of perimeter and area.

UNIT 9 INFORMATION HANDLING

9.1 Average	i)	Define an average (arithmetic mean).
	ii)	Find an average of given numbers.
	iii)	Solve real life problems involving average.
9.2 Block, Column and Bar Graphs	i)	Draw block graphs or column graphs.
	ii)	Read a simple bar graph given in horizontal and vertical form.
	iii)	Interpret a simple bar graph given in horizontal and vertical form.
	iv)	Define and organize a given data.

CURRICULUM FOR MATHEMATICS – GRADE VI

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 SETS

1.1 Set	i)	Define set. Recognize notation of a set and its objects/elements.
	ii)	Describe tabular form of a set and demonstrate through examples.
1.2 Types of Set		Define • finite and infinite sets, • empty/void/null set, • singleton, • equal and equivalent sets, • subset and superset of a set, • proper and improper subsets of a set, and demonstrate through examples.

UNIT 2 WHOLE NUMBERS

2.1 Natural and Whole Numbers	i)	Differentiate between natural and whole numbers.
	ii)	Identify natural and whole numbers, and their notations.
	iii)	Represent • a given list of whole numbers, • whole numbers $<$ (or $>$) a given whole number, • whole numbers $\geq$ (or $\leq$) a given whole number, • whole numbers $>$ but $<$ a given whole number, • whole numbers $\geq$ but $\leq$ a given whole number,

		• sum of two or more given whole numbers, on the number line.
2.2 Addition and Subtraction of Whole Numbers	i) ii) iii)	Add and subtract two given whole numbers. Verify commutative and associative law (under addition) of whole numbers. Recognize '0' as additive identity.
2.3 Multiplication and Division of Whole Numbers	i) ii) iii)	Multiply and divide two given whole numbers. Verify commutative and associative law (under multiplication) of whole numbers. Recognize '1' as multiplicative identity.
2.4 Multiplication and Addition (Subtraction) of Whole Numbers	i) ii)	Verify distributive law of multiplication over addition. Verify distributive law of multiplication over subtraction (with positive difference).

UNIT 3 FACTORS AND MULTIPLES

3.1 Factors and Multiples	i) ii) iii) iv) v) vi) vii)	Define a factor as a number which divides the dividend completely leaving no remainder. Define a multiple as a dividend into which a factor can divide. Define even numbers as the numbers which are multiples of 2. Define odd numbers as the numbers which are not multiples of 2. Define prime numbers as numbers which have only two factors (i.e., 1 and itself). Define composite numbers as numbers which have more than two factors. Know that 1 is neither prime nor composite as it has

		only one factor which is 1 itself.
	viii)	Know that 1 is a factor of every number.
	ix)	Know that 2 is the only even prime number whereas all other prime numbers are odd.
3.2 Tests for Divisibility		Test by inspection whether the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15 and 25 can divide a given number.
3.3 Factorization	i)	Define prime factorization as the process of factorizing a number into its prime factors.
	ii)	Recognize index notation.
	iii)	Factorize a given number and express its factors in the index notation.
3.4 HCF	i)	Define HCF as the greatest number which is a common factor of two or more numbers.
	ii)	Find HCF of two or more than two numbers by • prime factorization, • long division method.
3.5 LCM	i)	Define LCM as the smallest number which is a common multiple of two or more numbers.
	ii)	Find LCM of two or more numbers by • prime factorization, • division method.
3.6 Applications of HCF and LCM		Solve real life problems related to HCF and LCM.

UNIT 4 INTEGERS

4.1 Integers	i)	Know that • the natural numbers $1,2,3,\cdots$, are also called positive integers and the corresponding negative numbers $-1,-2,-3,\cdots$, are called negative integers, • '0' is an integer which is neither positive nor negative.
	ii)	Recognize integers.
4.2 Ordering of Integers	i)	Represent integers on number line.
	ii)	Know that on the number line any number lying • to the right of zero is positive, • to the left of zero is negative, • to the right of another number is greater, • to the left of another number is smaller.
	iii)	Know that every positive integer is greater than a negative integer.
	iv)	Know that every negative integer is less than a positive integer.
	v)	Arrange a given list of integers in ascending and descending order.
4.3 Absolute or Numerical Value of an Integer	i)	Define absolute or numerical value of a number as its distance from zero on the number line and is always positive.
	ii)	Arrange the absolute or numerical values of the given integers in ascending and descending order.
4.4 Addition of Integers	i)	Use number line to display: • sum of two or more given negative integers, • difference of two given positive integers,

	• sum of two given integers. ii) Add two integers (with like signs) in the following three steps: a) Take absolute values of given integers, b) Add the absolute values, c) Give the result the common sign. iii) Add two integers (with unlike signs) in the following three steps: a) Take absolute values of given integers, b) Subtract the smaller absolute value from the larger, c) Give the result the sign of the integer with the larger absolute value.
4.5 Subtraction of Integers	i) Recognize subtraction as the inverse process of addition. ii) Subtract one integer from the other by changing the sign of the integer being subtracted and adding according to the rules for addition of integers.
4.6 Multiplication of Integers	Recognize that • the product of two integers of like signs is a positive integer, • the product of two integers of unlike signs is a negative integer.
4.7 Division of Integers	i) Recognize that division is the inverse process of multiplication. ii) Recognize that on dividing one integer by another • if both the integers have like signs the quotient is positive, • if both the integers have unlike signs the quotient is negative. iii) Know that division of an integer by '0' is not possible.

UNIT 5 SIMPLIFICATIONS

5.1 BODMAS Rule	i)	Know that the following four kinds of brackets • $\overline{\quad}$ vinculum, • ( ) parentheses or curved brackets or round brackets, • { } braces or curly brackets, • [ ] square brackets or box brackets, are used to group two or more numbers together with operations.
	ii)	Know the order of preference as, $\overline{\quad}$, ( ), { } and [ ], to remove (simplify) them from an expression.
	iii)	Recognize BODMAS rule to follow the order in which the operations, to simplify mathematical expressions, are performed.
	iv)	Simplify mathematical expressions involving fractions and decimals grouped with brackets using BODMAS rule.
	v)	Solve real life problems involving fractions and decimals.

UNIT 6 RATIO AND PROPORTION

6.1 Ratio	i)	Define ratio as a relation which one quantity bears to another quantity of the same kind with regard to their magnitudes.
	ii)	Know that of the two quantities forming a ratio, the first one is called antecedent and the second one consequent.
	iii)	Know that a ratio has no units.
	iv)	Calculate ratio of two numbers.

	v) vi)	Reduce given ratio into lowest (equivalent) form. Describe the relationship between ratio and fraction.
6.2 Proportion	i) ii) iii)	Know that an equality of two ratios constitutes a proportion, e.g., $a:b::c:d$, where a, d are known as extremes and b, c are called the means. Find proportion (direct and inverse). Solve real life problems involving direct and inverse proportion.

UNIT 7 FINANCIAL ARITHMETIC

7.1 Percentage	i) ii) iii) iv) v) vi)	Recognize percentage as a fraction with denominator of 100. Convert a percentage to a fraction by expressing it as a fraction with denominator 100 and then simplify. Convert a fraction to a percentage by multiplying it with 100%. Convert a percentage to a decimal by expressing it as a fraction with denominator 100 and then as a decimal. Convert a decimal to a percentage by expressing it as a fraction with denominator 100 then as a percentage. Solve real life problems involving percentage.
7.2 Profit, Loss and Discount	i) • selling price and cost price, • profit, loss and discount, • profit percentage and loss percentage. ii)	Define Solve real life problems involving profit, loss and discount.

UNIT 8 INTRODUCTION TO ALGEBRA

8.1 Algebra	i) Explain the term algebra as an extension of arithmetic in which letters replace the numbers. ii) Know that • a sentence is a set of words making a complete grammatical structure and conveying full meaning. • sentences that are either true or false are known as statements. • a statement must be either true or false but not both. • a sentence that does not include enough information required to decide whether it is true or false is know as open statement (e.g., $\Delta + 2 = 9$). • a number that makes an open statement true is said to satisfy the statement (e.g. $\Delta = 7$ makes the statement $\Delta + 2 = 9$ true). • use English alphabet x in the open statement $\Delta + 2 = 9$ to modify it to $x + 2 = 9$. iii) Define variables as letters used to denote numbers in algebra. iv) Know that any numeral, variable or combination of numerals and variables connected by one or more of the symbols '+' and '–' is know as an algebraic expression (e.g., $x + 2y$).

8.2 Algebraic Expression	i)	Know that x, $2y$ and 5 are called the terms of the expression $x+2y+5$.
	ii)	Know that the symbol or number appearing as multiple of a variable used in algebraic term is called its coefficient (e.g. in $2y$, 2 is the coefficient of y).
	iii)	Know that the number, appearing in algebraic expression, independent of a variable is called a constant term (e.g. in $x+2y+5$, number 5 is a constant term).
	iv)	Differentiate between like and unlike terms.
	v)	Know that • like terms can be combined to give a single term, • addition or subtraction can not be performed with unlike terms.
	vi)	Add and subtract given algebraic expressions.
	vii)	Simplify algebraic expressions grouped with brackets.
	viii)	Evaluate and simplify an algebraic expression when the values of variables involved are given.

UNIT 9 LINEAR EQUATIONS

9.1 Algebraic Equations	i)	Define an algebraic equation.
	ii)	Differentiate between equation and an expression.
9.2 Linear Equations	i)	Define linear equation in one variable.
	ii)	Construct linear expression and linear equation in one variable.
	iii)	Solve simple linear equations involving fractional and decimal coefficients like $\frac{1}{2}x+5=x-\frac{1}{3}$.
	iv)	Solve real life problems involving linear equations.

UNIT 10 GEOMETRY

10.1 Line Segments	i)	Add measures of two or more line segments.
	ii)	Subtract measure of a line segment from a longer one.
	iii)	Draw a right bisector of a given line segment using compasses.
	iv)	Draw a perpendicular to a given line from a point on it using compasses.
	v)	Draw a perpendicular to a given line, from a point outside the line, using compasses.
10.2 Construction of Angles		Use compasses to • construct an angle equal in measure of a given angle, • construct an angle twice in measure of a given angle, • bisect a given angle, • divide a given angle into four equal angles, • construct the following angles: $60^\circ, 30^\circ, 15^\circ, 90^\circ, 45^\circ, (22\frac{1}{2})^\circ, 75^\circ, (67\frac{1}{2})^\circ, 120^\circ, 150^\circ$ $165^\circ, 135^\circ, 105^\circ$.
10.3 Construction of Triangles	i)	Construct a triangle when three sides (SSS) are given. **Caution**: Sum of two sides should be greater than the third side.
	ii)	Construct a triangle when two sides and their included angle (SAS) are given.
	iii)	Construct a triangle when two angles and the included side (ASA) are given.
	iv)	Construct a triangle when hypotenuse and one side (RHS) for a right angled triangle are given.

UNIT 11 PERIMETER AND AREA

11.1 Perimeter and Area	i)	Find perimeter and area of a square and a rectangle.
	ii)	Find area of path (inside or outside) of a rectangle or square.
	iii)	Solve real life problems related to perimeter and area of a square and rectangle.
	iv)	Recognize altitude of a geometric figure as the measure of the shortest distance between the base and its top.
	v)	Find area of a parallelogram when altitude and base are given.
	vi)	Define trapezium and find its area when altitude and measures of the parallel sides are given.
	vii)	Find area of a triangle when measures of the altitude and base are given.

UNIT 12 THREE DIMENSIONAL SOLIDS

12.1 Volume and Surface Area	i)	Identify 3D figure (cube, cuboid, sphere, cylinder and cone) with respect to their faces, edges and vertices.
	ii)	Define and recognize units of surface area and volume.
	iii)	Find surface area and volume of cube and cuboid.
	iv)	Solve real life problems involving volume and surface area.

UNIT 13 INFORMATION HANDLING

13.1 Types of Data	i)	Define data and data collection.
	ii)	Distinguish between grouped and ungrouped data.
13.2 Bar Graph		Draw horizontal and vertical bar graphs.
13.3 Pie Graph		Read a pie graph.

CURRICULUM FOR MATHEMATICS – GRADE VII

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 SETS

1.1 Set	i)	Express a set in • descriptive form, • set builder form, • tabular form.
1.2 Operations on Sets	i)	Define union, intersection and difference of two sets.
	ii)	Find • union of two or more sets, • intersection of two or more sets, • difference of two sets.
	iii)	Define and identify disjoint and overlapping sets.
	iv)	Define a universal set and complement of a set.
	v)	Verify different properties involving union of sets, intersection of sets, difference of sets and complement of a set, e.g., $A \cap A' = \phi$.
1.3 Venn Diagram	i)	Represent sets through Venn diagram.
	ii)	Perform operations of union, intersection, difference and complement on two sets A and B when • A is subset of B, • B is subset of A, • A and B are disjoint sets, • A and B are overlapping sets, through Venn diagram.

UNIT 2 RATIONAL NUMBERS

2.1 Rational Numbers	i)	Define a rational number as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q > 0$.
	ii)	Represent rational numbers on number line.
2.2 Operations on Rational Numbers	i)	Add two or more rational numbers.
	ii)	Subtract a rational number from another.
	iii)	Find additive inverse of a rational number.
	iv)	Multiply two or more rational numbers.
	v)	Divide a rational number by a non-zero rational number.
	vi)	Find multiplicative inverse of a rational number.
	vii)	Find reciprocal of a rational number.
	viii)	Verify commutative property of rational numbers with respect to addition and multiplication.
	ix)	Verify associative property of rational numbers with respect to addition and multiplication.
	x)	Verify distributive property of rational numbers with respect to multiplication over addition/ subtraction.
	xi)	Compare two rational numbers.
	xii)	Arrange rational numbers in ascending or descending order.

UNIT 3 DECIMALS

3.1 Conversion of Decimals to Rational Numbers		Convert decimals to rational numbers.
3.2 Terminating and Non-terminating Decimals	i)	Define terminating decimals as decimals having a finite number of digits after the decimal point.
	ii)	Define recurring decimals as non-terminating decimals in which a single digit or a block of digits

		repeats itself infinite number of times after decimal point (e.g. $\frac{2}{7} = 0 \cdot 285714285714285714\ldots$).
	iii)	Use the following rule to find whether a given rational number is terminating or not. Rule: If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal.
	iv)	Express a given rational number as a decimal and indicate whether it is terminating or recurring.
3.3 Approximate Value		Get an approximate value of a number, called rounding off, to a desired number of decimal places.

UNIT 4 EXPONENTS

4.1 Exponents/Indices 4.2 Laws of Exponents/Indices	i) ii)	Identify base, exponent and value. Use rational numbers to deduce laws of exponents. • Product Law: when bases are same but exponents are different: $a^m \times a^n = a^{m+n}$, when bases are different but exponents are same: $a^n \times b^n = (ab)^n$. • Quotient Law: when bases are same but exponents are different: $a^m \div a^n = a^{m-n}$, when bases are different but exponents are same: $a^n \div b^n = \left(\frac{a}{b}\right)^n$. • Power law: $\left(a^m\right)^n = a^{mn}$.

	• For zero exponent: $a^0 = 1$. • For exponent as negative integer: $a^{-m} = \frac{1}{a^m}$. iii) Demonstrate the concept of power of integer that is $(-a)^n$ when n is even or odd integer. iv) Apply laws of exponents to evaluate expressions.

UNIT 5 SQUARE ROOT OF POSITIVE NUMBER

5.1 Perfect Squares	i) Define a perfect square. ii) Test whether a number is a perfect square or not. iii) Identify and apply the following properties of perfect square of a number. • The square of an even number is even. • The square of an odd number is odd. • The square of a proper fraction is less than itself. • The square of a decimal less than 1 is smaller than the decimal.
5.2 Square Roots	i) Define square root of a natural number and recognize its notation. ii) Find square root, by division method and factorization method, of • natural number, • fraction, • decimal, which are perfect squares. iii) Solve real life problems involving square roots.

UNIT 6 DIRECT AND INVERSE VARIATION

6.1 Continued Ratio	i)	Define continued ratio and recall direct and inverse proportion.
	ii)	Solve real life problems (involving direct and inverse proportion) using unitary method and proportion method.
6.2 Time, Work and Distance	i)	Solve real life problems related to time and work using proportion.
	ii)	Find relation (i.e. speed) between time and distance.
	iii)	Convert units of speed (kilometer per hour into meter per second and vice versa).
	iv)	Solve variation related problems involving time and distance.

UNIT 7 FINANCIAL ARITHMETIC

7.1 Taxes	i)	Explain property tax and general sales tax.
	ii)	Solve tax-related problems.
7.2 Profit and Markup	i)	Explain profit and markup.
	ii)	Find the rate of profit/ markup per annum.
	iii)	Solve real life problems involving profit/ markup.
7.3 Zakat and Ushr	i)	Define zakat and ushr.
	ii)	Solve problems related to zakat and ushr.

UNIT 8 ALGEBRAIC EXPRESSIONS

8.1 Algebraic Expressions	i)	Define a constant as a symbol having a fixed numerical value.
	ii)	Recall variable as a quantity which can take various numerical values.

	iii)	Recall literal as an unknown number represented by an alphabet.
	iv)	Recall algebraic expression as a combination of constants and variables connected by the signs of fundamental operations.
	v)	Define polynomial as an algebraic expression in which the powers of variables are all whole numbers.
	vi)	Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms and three terms respectively.
8.2 Operations with Polynomials	i)	Add two or more polynomials.
	ii)	Subtract a polynomial from another polynomial.
	iii)	Find the product of • monomial with monomial, • monomial with binomial/trinomial, • binomials with binomial/trinomial.
	iv)	Simplify algebraic expressions involving addition, subtraction and multiplication.
8.3 Algebraic Identities		Recognize and verify the algebraic identities: • $(x+a)(x+b) = x^2 + (a+b)x + ab$, • $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$, • $(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$, • $a^2 - b^2 = (a-b)(a+b)$.
8.4 Factorization of Algebraic Expressions	i)	Factorize an algebraic expression (using algebraic identities).
	ii)	Factorize an algebraic expression (making groups).

UNIT 9 LINEAR EQUATIONS

9.1 Linear Equation	i)	Define a linear equation in one variable.
9.2 Solution of Linear Equation	i)	Demonstrate different techniques to solve linear equation.
	ii)	Solve linear equations of the type: • $ax+b=c$, • $\frac{ax+b}{cx+d}=\frac{m}{n}$.
	iii)	Solve real life problems involving linear equations.

UNIT 10 FUNDAMENTALS OF GEOMETRY

10.1 Properties of Angles	i)	Define adjacent, complementary and supplementary angles.
	ii)	Define vertically opposite angles.
	iii)	Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
	iv)	Calculate unknown angle of a triangle.
10.2 Congruent and Similar Figures	i)	Identify congruent and similar figures.
	ii)	Recognize the symbol of congruency.
	iii)	Apply the properties for two figures to be congruent or similar.
10.3 Congruent Triangles		Apply following properties for congruency between two triangles. • $SSS \cong SSS$, • $SAS \cong SAS$, • $ASA \cong ASA$, • $RHS \cong RHS$.

10.4 Circle	i)	Describe a circle and its centre, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
	ii)	Draw a semicircle and demonstrate the property; the angle in a semicircle is a right angle.
	iii)	Draw a segment of a circle and demonstrate the property; the angles in the same segment of a circle are equal.

UNIT 11 PRACTICAL GEOMETRY

11.1 Line Segment	i)	Divide a line segment into a given number of equal segments.
	ii)	Divide a line segment internally in a given ratio.
11.2 Triangles	i)	Construct a triangle when perimeter and ratio among the lengths of sides are given.
	ii)	Construct an equilateral triangle when • base is given, • altitude is given.
	iii)	Construct an isosceles triangle when • base and a base angle are given, • vertical angle and altitude are given, • altitude and a base angle are given.
11.3 Parallelogram	i)	Construct a parallelogram when • two adjacent sides and their included angle are given, • two adjacent sides and a diagonal are given.
	ii)	Verify practically that the sum of • measures of angles of a triangle is 180°. • measures of angles of a quadrilateral is 360°.

UNIT 12 CIRCUMFERENCE, AREA AND VOLUME

12.1 Circumference and Area of Circle	i)	Express π as the ratio between the circumference and the diameter of a circle.
	ii)	Find the circumference of a circle using formula.
	iii)	Find the area of a circular region using formula.
12.2 Surface Area and Volume of Cylinder	i)	Find the surface area of a cylinder using formula.
	ii)	Find the volume of a cylindrical region using formula.
	iii)	Solve real life problems involving • circumference and area of a circle, • surface area and volume of a cylinder.

UNIT 13 INFORMATION HANDLING

13.1 Frequency Distribution	i)	Demonstrate data presentation.
	ii)	Define frequency distribution (i.e. frequency, lower class limit, upper class limit, class interval).
13.2 Pie Graph		Interpret and draw pie graph.

CURRICULUM FOR MATHEMATICS – GRADE VIII

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 OPERATIONS ON SETS

1.1 Sets	i)	Recognize set of • natural numbers (N), • whole numbers (W), • integers (Z), • rational numbers (Q), • even numbers (E), • odd numbers (O), • prime numbers (P).
	ii)	Find a subset of a set.
	iii)	Define proper ($\subset$) and improper ($\subseteq$) subsets of a set.
	iv)	Find power set P(A) of a set A.
1.2 Operations on Sets	i)	Verify commutative and associative laws with respect to union and intersection.
	ii)	Verify the distributive laws.
	iii)	State and verify De Morgan's laws.
1.3 Venn Diagram	i)	Demonstrate union and intersection of three overlapping sets through Venn diagram.
	ii)	Verify associative and distributive laws through Venn diagram.

UNIT 2 REAL NUMBERS

2.1 Irrational Numbers	i)	Define an irrational number.
	ii)	Recognize rational and irrational numbers.
	iii)	Define real numbers.
	iv)	Demonstrate non-terminating /non-repeating (or non-periodic) decimals.
2.2 Squares	i)	Find perfect square of a number.
	ii)	Establish patterns for the squares of natural numbers (e.g., $4^2 = 1+2+3+4+3+2+1$).
2.3 Square Roots	i)	Find square root of • a natural number (e.g. 16, 625, 1600), • a common fraction (e.g. $\frac{9}{16}, \frac{36}{49}, \frac{49}{64}$), • a decimal (e.g. 0.01, 1.21, 0.64), given in perfect square form, by prime factorization and division method.
	ii)	Find square root of a number which is not a perfect square (e.g., the numbers 2, 3, 2.5).
	iii)	Use the following rule to determine the number of digits in the square root of a perfect square. Rule: Let n be the number of digits in the perfect square then its square root contains $\frac{n}{2}$ digits if n is even, $\frac{n+1}{2}$ digits if n is odd.
	iv)	Solve real life problems involving square roots.
2.4 Cubes and Cube Roots	i)	Recognize cubes and perfect cubes.
	ii)	Find cube roots of a number which are perfect cubes.
	iii)	Recognize properties of cubes of numbers.

UNIT 3 NUMBER SYSTEMS

3.1 Number System	i)	Recognize base of a number system.
	ii)	Define number system with base 2, 5, 8 and 10.
	iii)	Explain • binary number system (system with base 2), • number system with base 5, • octal number system (system with base 8), • decimal number system (system with base 10).
3.2 Conversions	i)	Convert a number from decimal system to a system with base 2, 5 and 8, and vice versa.
	ii)	Add, subtract and multiply numbers with base 2, 5 and 8.
	iii)	Add, subtract and multiply numbers with different bases.

UNIT 4 FINANCIAL ARITHMETIC

4.1 Compound Proportion	i)	Define compound proportion.
	ii)	Solve real life problems involving compound proportion, partnership and inheritance.
4.2 Banking		
4.2.1 Types of a Bank Account	i)	Define commercial bank deposits, types of a bank account (PLS savings bank account, current deposit account, PLS term deposit account and foreign currency account).
	ii)	Describe negotiable instruments like cheque, demand draft and pay order.
4.2.2 On-line banking	iii)	Explain on-line banking, transactions through ATM (Auto Teller Machine), debit card and credit card (Visa and Master).

4.2.3 Conversion of Currencies	iv)	Convert Pakistani currency to well-known international currencies.
4.2.4 Profit/ Markup	v)	Calculate • the profit/ markup, • the principal amount, • the profit/ markup rate, • the period.
4.2.5 Types of Finance	vi)	Explain • Overdraft (OD), • Running Finance (RF), • Demand Finance (DF), • Leasing.
	vii)	Solve real life problems related to banking and finance.
4.3 Percentage		
4.3.1 Profit and Loss	i)	Find percentage profit and percentage loss.
4.3.2 Discount	ii)	Find percentage discount.
	iii)	Solve problems involving successive transactions.
4.4 Insurance	i)	Define insurance.
	ii)	Solve real life problems regarding life and vehicle insurance.
4.5 Income Tax	i)	Explain income tax, exempt income and taxable income.
	ii)	Solve simple real life problems related to individual income tax assessee.

UNIT 5 POLYNOMIALS

5.1 Algebraic Expression	i)	Recall constant, variable, literal and algebraic expression.

5.2 Polynomial	i)	Define • polynomial, • degree of a polynomial, • coefficients of a polynomial.
	ii)	Recognize polynomial in one, two and more variables.
	iii)	Recognize polynomials of various degrees (e.g., linear, quadratic, cubic and biquadratic polynomials).
5.3 Operations on Polynomials	i)	Add, subtract and multiply polynomials.
	ii)	Divide a polynomial by a linear polynomial.

UNIT 6 FACTORIZATION, SIMULTANEOUS EQUATIONS

6.1 Basic Algebraic Formulas	Recall the formulas: • $(a+b)^2 = a^2 + 2ab + b^2$, • $(a-b)^2 = a^2 - 2ab + b^2$, • $a^2 - b^2 = (a-b)(a+b)$, and apply them to solve problems like: • Evaluate $(102)^2$, $(1.02)^2$, $(98)^2$ and $(0.98)^2$. • Find $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$ when the value of $x \pm \frac{1}{x}$ is given.
6.2 Factorization	Factorize expressions of the following types: • $ka + kb + kc$, • $ac + ad + bc + bd$, • $a^2 \pm 2ab + b^2$, • $a^2 - b^2$, • $a^2 \pm 2ab + b^2 - c^2$.
6.3 Manipulation of Algebraic	Recognize the formulas:

Expression	• $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, • $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$, and apply them to solve the problems like: • Find $x^3 + \frac{1}{x^3}$ and $x^3 - \frac{1}{x^3}$ when the value of $x \pm \frac{1}{x}$ is given.
6.4 Simultaneous Linear Equations	i) Recognize simultaneous linear equations in one and two variables. ii) Give the concept of formation of linear equation in two variables. iii) Know that: • a single linear equation in two unknowns is satisfied by as many pair of values as required. • two linear equations in two unknowns have only one solution (i.e., one pair of values).
6.5 Solution of Simultaneous Linear Equations	i) Solve simultaneous linear equations using • method of equating the coefficients, • method of elimination by substitution, • method of cross multiplication. ii) Solve real life problems involving two simultaneous linear equations in two variables.
6.6 Elimination	i) Eliminate a variable from two equations by: • Substitution, • application of formulae.
GUIDANCE FOR AUTHOR • For the elimination problems involving equations of motion use v_i for initial velocity and v_f for final velocity.	

UNIT 7 FUNDAMENTALS OF GEOMETRY

7.1 Parallel Lines	i)	Define parallel lines.
	ii)	Demonstrate through figures the following properties of parallel lines. • Two lines which are parallel to the same given line are parallel to each other. • If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal. • A line through the midpoint of the side of a triangle parallel to another side bisects the third side (an application of above property).
	iii)	Draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate interior angles, vertically opposite angles and interior angles on the same side of transversal.
	iv)	Describe the following relations between the pairs of angles when a transversal intersects two parallel lines. • Pairs of corresponding angles are equal. • Pairs of alternate interior angles are equal. • Pair of interior angles on the same side of transversal is supplementary, and demonstrate them through figures.
7.2 Polygons	i)	Define a polygon.
	ii)	Demonstrate the following properties of a parallelogram. • Opposite sides of a parallelogram are equal.

	• Opposite angles of a parallelogram are equal. • Diagonals of a parallelogram bisect each other. iii) Define regular pentagon, hexagon and octagon.
7.3 Circle	i) Demonstrate a point lying in the interior and exterior of a circle. ii) Describe the terms; sector, secant and chord of a circle, concyclic points, tangent to a circle and concentric circles.

UNIT 8 PRACTICAL GEOMETRY

8.1 Construction of Quadrilaterals	i) Define and depict two converging (non-parallel) lines and find the angle between them without producing the lines. ii) Bisect the angle between the two converging lines without producing them. iii) Construct a square • when its diagonal is given. • when the difference between its diagonal and side is given. • when the sum of its diagonal and side is given. iv) Construct a rectangle • when two sides are given. • when the diagonal and a side are given. v) Construct a rhombus • when one side and the base angle are given. • when one side and a diagonal are given. vi) Construct a parallelogram • when two diagonals and the angle between them is given.

	• when two adjacent sides and the angle included between them is given. vii) Construct a kite • when two unequal sides and a diagonal are given. viii) Construct a regular pentagon • when a side is given. ix) Construct a regular hexagon • when a side is given.
8.2 Construction of a Right Angled Triangle	Construct a right angled triangle • when hypotenuse and one side are given. • when hypotenuse and the vertical height from its vertex to the hypotenuse are given.

UNIT 9 AREAS AND VOLUMES

9.1 Pythagoras Theorem	i) State the Pythagoras theorem and give its informal proof. ii) Solve right angled triangles using Pythagoras theorem.
9.2 Hero's Formula	State and apply Hero's formula to find the areas of triangular and quadrilateral regions.
9.3 Surface Area and Volume	i) Find the surface area and volume of a sphere. ii) Find the surface area and volume of a cone. iii) Solve real life problems involving surface area and volume of sphere and cone.

UNIT 10 DEMONSTRATIVE GEOMETRY

10.1 Demonstrative geometry 10.1.1 Reasoning	i) Define demonstrative geometry. ii) Describe the basics of reasoning.

10.1.2 Axioms, Postulates and Theorem	iii) Describe the types of assumptions (axioms and postulates). iv) Describe parts of a proposition. v) Describe the meanings of a geometrical theorem, corollary and converse of a theorem.
10.2 Theorems	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) If a straight line stands on another straight line, the sum of measures of two angles so formed is equal to two right angles. ii) If the sum of measures of two adjacent angles is equal to two right angles, the external arms of the angles are in a straight line. iii) If two lines intersect each other, then the opposite vertical angles are congruent. iv) In any correspondence of two triangles, if two sides and included angle of one triangle are congruent to the corresponding sides and included angle of the other, the two triangles are congruent. v) If two sides of a triangle are congruent, then the angles opposite to these sides are congruent. vi) An exterior angle of a triangle is greater in measure than either of its opposite interior angles. vii) If a transversal intersects two lines such that the pair of alternate angles are congruent then the lines are parallel. viii) If a transversal intersects two parallel lines the alternate angles so formed are congruent. ix) The sum of measures of the three angles of a triangle is 180°.

UNIT 11 INTRODUCTION TO TRIGONOMETRY

11.1 Trigonometry 11.2 Trigonometric Ratios of Acute Angles	i) ii) iii) iv) v) vi)	Define trigonometry. Define trigonometric ratios of an acute angle. Find trigonometric ratios of acute angles (30°, 60° and 45°). Define trigonometric ratios of complementary angles. Solve right angled triangles using trigonometric ratios. Solve real life problems to find heights (avoid naming angle of elevation).

UNIT 12 INFORMATION HANDLING

12.1 Frequency Distribution	i) ii) iii)	Define frequency, frequency distribution. Construct frequency table. Construct a histogram representing frequency table.
12.2 Measures of Central Tendency	i) ii) iii)	Describe measures of central tendency. Calculate mean (average), weighted mean, median and mode for ungrouped data. Solve real life problems involving mean (average), weighted mean, median and mode.

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 MATRICES AND DETERMINANTS

1.1 Introduction to Matrices		Define • a matrix with real entries and relate its rectangular layout (formation) with real life, • rows and columns of a matrix, • the order of a matrix, • equality of two matrices.
1.2 Types of Matrices		Define and identify row matrix, column matrix, rectangular matrix, square matrix, zero/null matrix, identity matrix, scalar matrix, diagonal matrix, transpose of a matrix, symmetric and skew-symmetric matrices.
1.3 Addition and Subtraction of Matrices	i) ii) iii) iv) v) vi)	Know whether the given matrices are conformable for addition/subtraction. Add and subtract matrices. Multiply a matrix by a real number. Verify commutative and associative laws under addition. Define additive identity of a matrix. Find additive inverse of a matrix.
1.4 Multiplication of Matrices	i) ii) iii) iv)	Know whether the given matrices are conformable for multiplication. Multiply two (or three) matrices. Verify associative law under multiplication. Verify distributive laws.

	v)	Show with the help of an example that commutative law under multiplication does not hold in general (i.e. $AB \neq BA$).
	vi)	Define multiplicative identity of a matrix.
	vii)	Verify the result $(AB)^t = B^t A^t$.
1.5 Multiplicative Inverse of a Matrix	i)	Define the determinant of a square matrix.
	ii)	Evaluate determinant of a matrix.
	iii)	Define singular and non-singular matrices.
	iv)	Define adjoint of a matrix.
	v)	Find multiplicative inverse of a non-singular matrix A and verify that $AA^{-1} = I = A^{-1}A$ where I is the identity matrix.
	vi)	Use adjoint method to calculate inverse of a non-singular matrix.
	vii)	Verify the result $(AB)^{-1} = B^{-1}A^{-1}$.
1.6 Solution of Simultaneous Linear Equations		Solve a system of two linear equations and related real life problems in two unknowns using • Matrix inversion method, • Cramer's rule.

GUIDANCE FOR AUTHOR

- Order of a matrix may be written as m–by–n instead of $m \times n$.
- A matrix of order at most 3–by–3 may be used while defining (considering) different types of matrices, their addition and scalar multiplication etc.
- Order of the matrix may be restricted to 2–by–2 when matrix multiplication, determinant, adjoint, inverse and system of equations are considered.

UNIT 2 REAL AND COMPLEX NUMBERS

2.1 Real Numbers	i)	Recall the set of real numbers as a union of sets of

		rational and irrational numbers. ii) Depict real numbers on the number line. iii) Demonstrate a number with terminating and non-terminating recurring decimals on the number line. iv) Give decimal representation of rational and irrational numbers.
2.2 Properties of Real Numbers		Know the properties of real numbers.
2.3 Radicals and Radicands	i) ii) iii)	Explain the concept of radicals and radicands. Differentiate between radical form and exponential form of an expression. Transform an expression given in radical form to an exponential form and vice versa.
2.4 Laws of Exponents/Indices	i) ii)	Recall base, exponent and value. Apply the laws of exponents to simplify expressions with real exponents.
2.5 Complex Numbers	i) ii) iii) iv)	Define complex number z represented by an expression of the form $z = a + ib$, where a and b are real numbers and $i = \sqrt{-1}$. Recognize a as real part and b as imaginary part of $z = a + ib$. Define conjugate of a complex number. Know the condition for equality of complex numbers.
2.6 Basic Operations on Complex Numbers		Carryout basic operations (i.e. addition, subtraction, multiplication and division) on complex numbers.

UNIT 3 LOGARITHMS

3.1 Scientific Notation		Express a number in standard form of scientific notation and vice versa.

3.2 Logarithm	i) ii) iii) iv)	Define logarithm of a number to the base a as the power to which a must be raised to give the number (i.e. $a^x = y \Leftrightarrow \log_a y = x,\ a > 0,\ y > 0$ and $a \neq 1$). Define a common logarithm, characteristic and mantissa of log of a number. Use tables to find the log of a number. Give concept of antilog and use tables to find the antilog of a number.
3.3 Common and Natural Logarithm		Differentiate between common and natural logarithm.
3.4 Laws of Logarithm		Prove the following laws of logarithm. • $\log_a(mn) = \log_a m + \log_a n$, • $\log_a(\frac{m}{n}) = \log_a m - \log_a n$, • $\log_a m^n = n \log_a m$, • $\log_a m \log_m n = \log_a n$.
3.5 Application of Logarithm		Apply laws of logarithm to convert lengthy processes of multiplication, division and exponentiation into easier processes of addition and subtraction etc.

UNIT 4 ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

4.1 Algebraic expressions	i) ii) iii)	Know that a rational expression behaves like a rational number. Define a rational expression as the quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$ where $q(x)$ is not the zero polynomial. Examine whether a given algebraic expression is a • polynomial or not, • rational expression or not.

	iv)	Define $\frac{p(x)}{q(x)}$ as a rational expression in its lowest terms if $p(x)$ and $q(x)$ are polynomials with integral coefficients and having no common factor.
	v)	Examine whether a given rational algebraic expression is in lowest form or not.
	vi)	Reduce a given rational expression to its lowest terms.
	vii)	Find the sum, difference and product of rational expressions.
	viii)	Divide a rational expression with another and express the result in its lowest terms.
	ix)	Find value of algebraic expression at some particular real number.
4.2 Algebraic Formulae	i)	Know the formulas $(a+b)^2+(a-b)^2=2(a^2+b^2)$, $(a+b)^2-(a-b)^2=4ab$. • Find the value of a^2+b^2 and of ab when the values of $a+b$ and $a-b$ are known.
	ii)	Know the formula $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$. • Find the value of $a^2+b^2+c^2$ when the values of $a+b+c$ and $ab+bc+ca$ are given. • Find the value of $a+b+c$ when the values of $a^2+b^2+c^2$ and $ab+bc+ca$ are given. • Find the value of $ab+bc+ca$ when the values of $a^2+b^2+c^2$ and $a+b+c$ are given.
	iii)	Know the formulas $(a+b)^3=a^3+3ab(a+b)+b^3$, $(a-b)^3=a^3-3ab(a-b)-b^3$.

		• Find the value of $a^3 \pm b^3$ when the values of $a \pm b$ and ab are given. • Find the value of $x^3 \pm \frac{1}{x^3}$ when the value of $x \pm \frac{1}{x}$ is given.
	iv)	Know the formula $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$. • Find the product of $x + \frac{1}{x}$ and $x^2 + \frac{1}{x^2} - 1$. • Find the product of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2} + 1$. • Find the continued product of $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$.
4.3 Surds and their Application	i) ii)	Recognize the surds and their application. Explain the surds of second order. Use basic operations on surds of second order to rationalize the denominators and evaluate it.
4.4 Rationalization		Explain rationalization (with precise meaning) of real numbers of the types $\frac{1}{a+b\sqrt{x}}$, $\frac{1}{\sqrt{x}+\sqrt{y}}$ and their combinations where x and y are natural numbers and a and b are integers

UNIT 5 FACTORIZATION

5.1 Factorization	Recall factorization of expressions of the following types. • $ka + kb + kc$, • $ac + ad + bc + bd$, • $a^2 \pm 2ab + b^2$, • $a^2 - b^2$, • $a^2 \pm 2ab + b^2 - c^2$.

	GUIDANCE FOR AUTHOR Include an exercise covering all above types of factorization.
	Factorize the expressions of the following types: Type I: $a^4+a^2b^2+b^4$ or a^4+4b^4, Type II: x^2+px+q, Type III: ax^2+bx+c, Type IV: $\begin{cases}(ax^2+bx+c)(ax^2+bx+d)+k,\\(x+a)(x+b)(x+c)(x+d)+k,\\(x+a)(x+b)(x+c)(x+d)+kx^2,\end{cases}$ Type V: $\begin{cases}a^3+3a^2b+3ab^2+b^3,\\a^3-3a^2b+3ab^2-b^3,\end{cases}$ Type VI: $a^3\pm b^3$.
5.2 Remainder Theorem and Factor Theorem	i) State and prove Remainder theorem and explain through examples. ii) Find remainder (without dividing) when a polynomial is divided by a linear polynomial. iii) Define zeros of a polynomial. iv) State and prove factor theorem.
5.3 Factorization of a Cubic Polynomial	Use factor theorem to factorize a cubic polynomial.

UNIT 6 ALGEBRAIC MANIPULATION

6.1 Highest Common Factor and Least Common Multiple	i) Find highest common factor and least common multiple of algebraic expressions. ii) Use factor or division method to determine highest common factor and least common multiple. iii) Know the relationship between HCF and LCM. iv) Solve real life problems related to HCF and LCM.

6.2 Basic Operations on Algebraic Fractions		Use highest common factor and least common multiple to reduce fractional expressions involving $+,-,\times,\div$.
6.3 Square Root of Algebraic Expression		Find square root of algebraic expression by factorization and division.

UNIT 7 LINEAR EQUATIONS AND INEQUALITIES

7.1 Linear Equations	i) ii) iii)	Recall linear equation in one variable. Solve linear equation with rational coefficients. Reduce equations, involving radicals, to simple linear form and find their solutions.
7.2 Equation involving Absolute Value	i) ii)	Define absolute value. Solve the equation, involving absolute value, in one variable.
7.3 Linear Inequalities	i) ii)	Define inequalities $(>, <)$ and $(\geq, \leq)$. Recognize properties of inequalities (i.e. trichotomy, transitive, additive and multiplicative).
7.4 Solving Linear Inequalities		Solve linear inequalities with rational coefficients.

UNIT 8 QUADRATIC EQUATIONS

8.1 Quadratic Equation		Define quadratic equation.
8.2 Solution of Quadratic Equations		Solve a quadratic equation in one variable by • factorization, • completing square.
8.3 Quadratic Formula	i) ii)	Use method of completing square to derive quadratic formula. Use quadratic formula to solve quadratic equations.

8.4 Equations Reducible to Quadratic Form	i)	Solve equations, reducible to quadratic form, of the type $ax^4 + bx^2 + c = 0$.
	ii)	Solve the equations of the type $a\,p(x) + \frac{b}{p(x)} = c$.
	iii)	Solve reciprocal equations of the type $a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$.
	iv)	Solve exponential equations in which the variables occur in exponents.
	v)	Solve equations of the type $(x+a)(x+b)(x+c)(x+d) = k$ where $a+b = c+d$.
8.5 Radical Equations		Solve equations of the type: • $\sqrt{ax+b} = cx+d$, • $\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$, • $\sqrt{x^2+px+m} + \sqrt{x^2+px+n} = q$.

UNIT 9 THEORY OF QUADRATIC EQUATIONS

9.1 Nature of the Roots of a Quadratic Equation	i)	Define discriminant $(b^2 - 4ac)$ of the quadratic expression $ax^2 + bx + c$.
	ii)	Find discriminant of a given quadratic equation.
	iii)	Discuss the nature of roots of a quadratic equation through discriminant.
	iv)	Determine the nature of roots of a given quadratic equation and verify the result by solving the equation.
	v)	Determine the value of an unknown involved in a given quadratic equation when the nature of its roots is given.
9.2 Cube Roots of Unity and	i)	Find cube roots of unity.

their Properties	ii)	Recognize complex cube roots of unity as ω and ω^2.
	iii)	Prove the properties of cube roots of unity.
	iv)	Use properties of cube roots of unity to solve appropriate problems.
9.3 Roots and Coefficients of a Quadratic Equation	i)	Find the relation between the roots and the coefficients of a quadratic equation.
	ii)	Find the sum and product of roots of a given quadratic equation without solving it.
	iii)	Find the value(s) of unknown(s) involved in a given quadratic equation when • sum of roots is equal to a multiple of the product of roots, • sum of the squares of roots is equal to a given number, • roots differ by a given number, • roots satisfy a given relation (e.g. the relation $2\alpha+5\beta=7$ where α and β are the roots of given equation), • both sum and product of roots are equal to a given number.
9.4 Symmetric Functions of Roots of a Quadratic Equation	i)	Define symmetric functions of roots of a quadratic equation.
	ii)	Evaluate a symmetric function of the roots of a quadratic equation in terms of its coefficients.
9.5 Formation of Quadratic Equation	i)	Establish the formula, $x^2-(\text{Sum of roots})x+(\text{Product of roots})=0$, to find a quadratic equation from the given roots.
	ii)	Form the quadratic equation whose roots, for

		example, are of the type: • $2\alpha+1, 2\beta+1,$ • $\alpha^2, \beta^2,$ • $\frac{1}{\alpha}, \frac{1}{\beta},$ • $\frac{\alpha}{\beta}, \frac{\beta}{\alpha},$ • $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta},$ where α, β are the roots of a given quadratic equation.
9.6 Synthetic Division	i) ii)	Describe the method of synthetic division. Use synthetic division to • find quotient and remainder when a given polynomial is divided by a linear polynomial, • find the value(s) of unknown(s) if the zeros of a polynomial are given, • find the value(s) of unknown(s) if the factors of a polynomial are given, • solve a cubic equation if one root of the equation is given, • solve a biquadratic (quartic) equation if two of the real roots of the equation are given.
9.7 Simultaneous Equations		Solve a system of two equations in two variables when • one equation is linear and the other is quadratic, • both the equations are quadratic.
		Solve the real life problems leading to quadratic equations.

UNIT 10 VARIATIONS

10.1 Ratio, Proportions and Variations	i) ii)	Define ratio, proportions and variations (direct and inverse). Find 3^{rd}, 4^{th} mean and continued proportion.
10.2 Theorems on Proportions		Apply theorems of invertendo, alternendo, componendo, dividendo and componendo & dividendo to find proportions.
10.3 Joint Variation	i) ii)	Define joint variation. Solve problems related to joint variation.
10.4 K-Method	i) ii)	Use K-Method to prove conditional equalities involving proportions. Solve real life problems based on variations.

UNIT 11 PARTIAL FRACTIONS

11.1 Proper, Improper and Rational Fraction	Define proper, improper and rational fraction.
11.2 Resolution of Fraction into Partial Fractions	Resolve an algebraic fraction into partial fractions when its denominator consists of • non-repeated linear factors, • repeated linear factors, • non-repeated quadratic factors, • repeated quadratic factors.

GUIDANCE FOR AUTHOR

In the denominator for

- non-repeated linear case take only two factors,
- repeated linear case take only two factors like $(x+a)^2(x+b)$,
- non-repeated quadratic case take only two factors like $(x+a)(x^2+b)$,
- repeated quadratic case take only two factors like $(x+a)(x^2+b)^2$.

Caution: Power of any factor should not exceed 2.

UNIT 12 SETS AND FUNCTIONS

12.1 Sets		
12.1.1 Operations on Sets	i)	Recall the sets denoted by N, Z, W, E, O, P and Q.
	ii)	Recognize operation on sets ($\cup$, $\cap$, \, …).
	iii)	Perform operation on sets • union, • intersection, • difference, • complement.
12.1.2 Properties of Union and Intersection	iv)	Give formal proofs of the following fundamental properties of union and intersection of two or three sets. • Commutative property of union, • Commutative property of intersection, • Associative property of union, • Associative property of intersection, • Distributive property of union over intersection, • Distributive property of intersection over union, • De Morgan's laws.
	v)	Verify the fundamental properties for given sets.
12.1.3 Venn Diagram	vi)	Use Venn diagram to represent • union and intersection of sets, • complement of a set.
	vii)	Use Venn diagram to verify • commutative law for union and intersection of sets, • De Morgan's laws, • associative laws, • distributive laws.
12.1.4 Ordered Pairs and Cartesian Product	viii)	Recognize ordered pairs and Cartesian product.

12.2 Binary relation		Define binary relation and identify its domain and range.
12.3 Function	i)	Define function and identify its domain, co-domain and range.
	ii)	Demonstrate the following: • into function, • one-one function, • into and one-one function (injective function), • onto function (surjective function), • one-one and onto function (bijective function).
	iii)	Examine whether a given relation is a function or not.
	iv)	Differentiate between one-one correspondence and one-one function.
	v)	Include sufficient exercises to clarify/differentiate between the above concepts.

UNIT 13 BASIC STATISTICS

13.1 Frequency Distribution	i)	Construct grouped frequency table.
	ii)	Construct histograms with equal and unequal class intervals.
	iii)	Construct a frequency polygon.
13.2 Cumulative Frequency Distribution	i)	Construct a cumulative frequency table.
	ii)	Draw a cumulative frequency polygon.
13.3 Measures of Central Tendency	i)	Calculate (for ungrouped and grouped data): • arithmetic mean by definition and using deviations from assumed mean, • median, mode, geometric mean, harmonic mean.
	ii)	Recognize properties of arithmetic mean.

	iii) iv)	Calculate weighted mean and moving averages. Estimate median, quartiles and mode graphically.
13.4 Measures of Dispersion		Measure range, variance and standard deviation.

UNIT 14 LINEAR GRAPHS AND THEIR APPLICATION

14.1 Cartesian Plane and Linear Graphs	i) ii) iii) iv) v) vi) vii) viii) ix) x)	Identify pair of real numbers as an ordered pair. Recognize an ordered pair through different examples; for instance an ordered pair (2,3) to represent a seat, located in an examination hall, at the 2nd row and 3rd column. Describe rectangular or Cartesian plane consisting of two number lines intersecting at right angles at the point O. Identify origin (O) and coordinate axes (horizontal and vertical axes or *x*-axis and *y*-axis) in the rectangular plane. Locate an ordered pair (*a*, *b*) as a point in the rectangular plane and recognize: • *a* as the *x*-coordinate (or abscissa), • *b* as the *y*-coordinate (or ordinate). Draw different geometrical shapes (e.g., line segment, triangle and rectangle etc) by joining a set of given points. Construct a table for pairs of values satisfying a linear equation in two variables. Plot the pairs of points to obtain the graph of a given expression. Choose an appropriate scale to draw a graph. Draw the graph of

		• an equation of the form $y = c$. • an equation of the form $x = a$. • an equation of the form $y = mx$. • an equation of the form $y = mx + c$.
	xi)	Draw a graph from a given table of (discrete) values.
	xii)	Solve appropriate real life problems.
14.2 Conversion Graphs	i)	Interpret conversion graph as a linear graph relating to two quantities which are in direct proportion.
	ii)	Read a given graph to know one quantity corresponding to another.
	iii)	Read the graph for conversions of the form: • miles and kilometers, • acres and hectares, • degrees Celsius and degrees Fahrenheit, • Pakistani currency and another currency, etc.
14.3 Graphic Solution of Equations in two Variables		Solve simultaneous linear equations in two variables using graphical method.

UNIT 15 INTRODUCTION TO COORDINATE GEOMETRY

15.1 Distance Formula	i)	Define coordinate geometry.
	ii)	Derive distance formula to calculate distance between two points given in Cartesian plane.
	iii)	Use distance formula to find distance between two given points.
15.2 Collinear Points	i)	Define collinear points. Distinguish between collinear and non-collinear points.
	ii)	Use distance formula to show that given three (or more) points are collinear.

	iii)	Use distance formula to show that the given three non-collinear points form: • an equilateral triangle, • an isosceles triangle, • a right angled triangle, • a scalene triangle.
	iv)	Use distance formula to show that given four non-collinear points form: • a square, • a rectangle, • a parallelogram.
15.3 Mid-point Formula	i)	Recognize the formula to find the midpoint of the line joining two given points.
	ii)	Apply distance and mid point formulae to solve/verify different standard results related to geometry.

UNIT 16 INTRODUCTION TO TRIGONOMETRY

16.1 Measurement of an Angle	i)	Measure an angle in sexagesimal system (degree, minute and second).
	ii)	Convert an angle given in $D^\circ M' S''$ form into a decimal form and vice versa.
	iii)	Define a radian (measure of an angle in circular system) and prove the relationship between radians and degrees.
16.2 Sector of a Circle	i)	Establish the rule $l = r\theta$, where r is the radius of the circle, l the length of circular arc and θ the central angle measured in radians.
	ii)	Prove that the area of a sector of a circle is $\frac{1}{2}r^2\theta$.

16.3 Trigonometric Ratios	i) Define and identify: • general angle (coterminal angles), • angle in standard position. ii) Recognize quadrants and quadrantal angles. iii) Define trigonometric ratios and their reciprocals with the help of a unit circle. iv) Recall the values of trigonometric ratios for $45^\circ, 30^\circ, 60^\circ$. v) Recognize signs of trigonometric ratios in different quadrants. vi) Find the values of remaining trigonometric ratios if one trigonometric ratio is given. vii) Calculate the values of trigonometric ratios for $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$.
16.4 Trigonometric Identities	Prove the trigonometric identities and apply them to show different trigonometric relations.
16.5 Angle of Elevation and Depression.	i) Find angle of elevation and depression. ii) Solve real life problems involving angle of elevation and depression.

UNIT 17 CONGRUENT TRIANGLES

17.1 Congruent Triangles	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, the two triangles are congruent. ii) If two angles of a triangle are congruent then the sides opposite to them are also congruent.

	iii) In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, the two triangles are congruent. iv) If in the correspondence of two right-angled triangles, the hypotenuse and one side of one are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent.

UNIT 18 PARALLELOGRAMS AND TRIANGLES

18.1 Parallelograms and Triangles	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) In a parallelogram: • the opposite sides are congruent, • the opposite angles are congruent, • the diagonals bisect each other. ii) If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram. iii) The line segment, joining the midpoints of two sides of a triangle, is parallel to the third side and is equal to one half of its length. iv) The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median. v) If three or more parallel lines make congruent intercepts on a transversal they also intercept congruent segments on any other line that cuts them.

UNIT 19 LINE BISECTORS AND ANGLE BISECTORS

19.1 Line Bisectors and Angle Bisectors	Prove the following theorems along with corollaries and apply them to solve appropriate problems.

	i) Any point on the right bisector of a line segment is equidistant from its end points. ii) Any point equidistant from the points of a line segment is on the right bisector of it. iii) The right bisectors of the sides of a triangle are concurrent. iv) Any point on the bisector of an angle is equidistant from its arms. v) Any point inside an angle, equidistant from its arms, is on the bisector of it. vi) The bisectors of the angles of a triangle are concurrent.

UNIT 20 SIDES AND ANGLES OF A TRIANGLE

20.1 Sides and Angles of a Triangle	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it. ii) If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle. iii) The sum of the lengths of any two sides of a triangle is greater than the length of the third side. iv) From a point, out-side a line, the perpendicular is the shortest distance from the point to the line.

UNIT 21 RATIO AND PROPORTION

21.1 Ratio and Proportion	Prove the following theorems along with corollaries and

	apply them to solve appropriate problems. i) A line parallel to one side of a triangle, intersecting the other two sides, divides them proportionally. ii) If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the third side. iii) The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the angle. iv) If two triangles are similar, the measures of their corresponding sides are proportional.

UNIT 22 PYTHAGORAS' THEOREM

22.1 Pythagoras' Theorem	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides. (Pythagoras' theorem). ii) If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a right angled triangle (converse to Pythagoras' theorem).

UNIT 23 THEOREMS RELATED WITH AREA

23.1 Theorems Related with Area	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) Parallelograms on the same base and lying between the same parallel lines (or of the same altitude) are equal in area.

	ii) Parallelograms on equal bases and having the same altitude are equal in area. iii) Triangles on the same base and of the same altitude are equal in area. iv) Triangles on equal bases and of the same altitude are equal in area.

UNIT 24 PROJECTION OF A SIDE OF A TRIANGLE

24.1 Projection of a Side of a Triangle	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) In an obtuse-angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other. ii) In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other. iii) In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side (Apollonius' Theorem).

UNIT 25 CHORDS OF A CIRCLE

25.1 Chords of a Circle	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) One and only one circle can pass through three non-

	collinear points. ii) A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord. iii) Perpendicular from the centre of a circle on a chord bisects it. iv) If two chords of a circle are congruent then they will be equidistant from the centre. v) Two chords of a circle which are equidistant from the centre are congruent.

UNIT 26 TANGENT TO A CIRCLE

26.1 Tangent to a Circle	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point. ii) The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other. iii) The two tangents drawn to a circle from a point outside it, are equal in length. iv) If two circles touch externally or internally, the distance between their centres is respectively equal to the sum or difference of their radii.

UNIT 27 CHORDS AND ARCS

27.1 Chords and Arcs	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) If two arcs of a circle (or of congruent circles) are

	congruent then the corresponding chords are equal. ii) If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent. iii) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres). iv) If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, the chords are equal.

UNIT 28 ANGLE IN A SEGMENT OF A CIRCLE

28.1 Angle in a Segment of a Circle	Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc. ii) Any two angles in the same segment of a circle are equal. iii) The angle • in a semi-circle is a right angle, • in a segment greater than a semi circle is less than a right angle, • in a segment less than a semi-circle is greater than a right angle. iv) The opposite angles of any quadrilateral inscribed in a circle are supplementary.

UNIT 29 PRACTICAL GEOMETRY – TRIANGLES

29.1 Construction of Triangle	i)	Construct a triangle having given: • two sides and the included angle, • one side and two of the angles, • two of its sides and the angle opposite to one of them (with all the three possibilities).
	ii)	Draw: • angle bisectors, • altitudes, • perpendicular bisectors, • medians, of a given triangle and verify their concurrency.
29.2 Figures with Equal Areas	i)	Construct a triangle equal in area to a given quadrilateral.
	ii)	Construct a rectangle equal in area to a given triangle.
	iii)	Construct a square equal in area to a given rectangle.
	iv)	Construct a triangle of equivalent area on a base of given length.

UNIT 30 PRACTICAL GEOMETRY – CIRCLES

30.1 Construction of Circle	i)	Locate the centre of a given circle.
	ii)	Draw a circle passing through three given non-collinear points.
	iii)	Complete the circle: • by finding the centre, • without finding the centre, when a part of its circumference is given.
30.2 Circles attached to	i)	Circumscribe a circle about a given triangle.

Polygons	ii) Inscribe a circle in a given triangle. iii) Escribe a circle to a given triangle. iv) Circumscribe an equilateral triangle about a given circle. v) Inscribe an equilateral triangle in a given circle. vi) Circumscribe a square about a given circle. vii) Inscribe a square in a given circle. viii) Circumscribe a regular hexagon about a given circle. ix) Inscribe a regular hexagon in a given circle.
30.3 Tangent to the Circle	i) Draw a tangent to a given arc, without using the centre, through a given point *P* when *P* is • the middle point of the arc, • at the end of the arc, • outside the arc. ii) Draw a tangent to a given circle from a point *P* when *P* lies • on the circumference, • outside the circle. iii) Draw two tangents to a circle meeting each other at a given angle. iv) Draw • direct common tangent or external tangent, • transverse common tangent or internal tangent to two equal circles. v) Draw • direct common tangent or external tangent, • transverse common tangent or internal tangent to two unequal circles. vi) Draw a tangent to • two unequal touching circles,

	• two unequal intersecting circles. vii) Draw a circle which touches • both the arms of a given angle, • two converging lines and passes through a given point between them, • three converging lines.

CURRICULUM FOR MATHEMATICS – GRADE XI

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 COMPLEX NUMBERS

1.1 Complex Numbers	i)	Recall complex number z represented by an expression of the form $z = a + ib$ or of the form (a,b) where a and b are real numbers and $i = \sqrt{-1}$.
	ii)	Recognize a as real part of z and b as imaginary part of z.
	iii)	Know the condition for equality of complex numbers.
	iv)	Carryout basic operations on complex numbers.
	v)	Define $\bar{z} = a - ib$ as the complex conjugate of $z = a + ib$.
	vi)	Define $\lvert z \rvert = \sqrt{a^2 + b^2}$ as the absolute value or modulus of a complex number $z = a + ib$.
1.2 Properties of Complex Numbers	i)	Describe algebraic properties of complex numbers (e.g. commutative, associative and distributive) with respect to '+' and '×'.
	ii)	Know additive identity and multiplicative identity for the set of complex numbers.
	iii)	Find additive inverse and multiplicative inverse of a complex number z.
	iv)	Demonstrate the following properties • $\lvert z \rvert = \lvert -z \rvert = \lvert \bar{z} \rvert = \lvert -\bar{z} \rvert$, • $\bar{\bar{z}} = z$, $z\,\bar{z} = \lvert z \rvert^2$, $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$,

		• $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$, $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$.
	v)	Find real and imaginary parts of the following type of complex numbers • $(x+iy)^n$, • $\left(\frac{x_1+iy_1}{x_2+iy_2}\right)^n$, $x_2+iy_2 \neq 0$, where $n = \pm 1$ and ± 2.
1.3 Solution of equations	i)	Solve the simultaneous linear equations with complex coefficients. For example, $\begin{cases} 5z-(3+i)w = 7-i, \\ (2-i)z+2iw = -1+i. \end{cases}$
	ii)	Write the polynomial $P(z)$ as a product of linear factors. For example, $z^2 + a^2 = (z+ia)(z-ia)$, $z^3 - 3z^2 + z + 5 = (z+1)(z-2-i)(z-2+i)$.
	iii)	Solve quadratic equation of the form $pz^2 + qz + r = 0$ by completing squares, where *p, q, r* are real numbers and *z* a complex number. For example: Solve $z^2 - 2z + 5 = 0$. $\Rightarrow (z-1-2i)(z-1+2i) = 0$, $\Rightarrow z = 1+2i, 1-2i$.

UNIT 2 MATRICES AND DETERMINANTS

2.1 Matrices	i)	Recall the concept of • a matrix and its notation, • order of a matrix, • equality of two matrices.

	ii)	Define row matrix, column matrix, square matrix, rectangular matrix, zero/null matrix, identity matrix, scalar matrix, diagonal matrix, upper and lower triangular matrix, transpose of a matrix, symmetric matrix and skew-symmetric matrix.
2.2 Algebra of Matrices	i)	Carryout scalar multiplication, addition/subtraction of matrices, multiplication of matrices with real and complex entries.
	ii)	Show that commutative property • holds under addition. • does not hold under multiplication, in general.
	iii)	Verify that $(AB)^t = B^t A^t$.
2.3 Determinants	i)	Describe determinant of a square matrix, minor and cofactor of an element of a matrix.
	ii)	Evaluate determinant of a square matrix using cofactors.
	iii)	Define singular and non-singular matrices.
	iv)	Know the adjoint of a square matrix.
	v)	Use adjoint method to calculate inverse of a square matrix.
	vi)	Verify the result $(AB)^{-1} = B^{-1}A^{-1}$.
2.4 Properties of Determinants	i)	State and prove the properties of determinants.
	ii)	Evaluate the determinant without expansion (i.e. using properties of determinants).
2.5 Row and Column Operations	i)	Know the row and column operations on matrices.
	ii)	Define echelon and reduced echelon form of a matrix.
	iii)	Reduce a matrix to its echelon and reduced echelon form.
	iv)	Recognize the rank of a matrix.

	v)	Use row operations to find the inverse and the rank of a matrix.
2.6 Solving System of Linear Equations	i)	Distinguish between homogeneous and non-homogeneous linear equations in 2 and 3 unknowns.
	ii)	Solve a system of three homogeneous linear equations in three unknowns.
	iii)	Define a consistent and inconsistent system of linear equations and demonstrate through examples.
	iv)	Solve a system of 3 by 3 non-homogeneous linear equations using: • matrix inversion method, • Gauss elimination method (echelon form), • Gauss-Jordan method (reduced echelon form), • Cramer's rule.
GUIDANCE FOR AUTHOR • At secondary level the notation for order of a matrix is supposed to be used as *m*–by–*n* whereas for this level both the notations *m*–by–*n* and $m \times n$ may be used for order of a matrix. • Formal proof of Gauss elimination method may be excluded. • Order of a matrix should not exceed 3–by–3.		

UNIT 3 VECTORS

3.1 Vectors in Plane	i)	Define a scalar and a vector.
	ii)	Give geometrical representation of a vector.
	iii)	Give the following fundamental definitions using geometrical representation: • magnitude of a vector, • equal vectors, • negative of a vector,

	• unit vector, • zero/null vector, • position vector, • parallel vectors, • addition and subtraction of vectors, • triangle, parallelogram and polygon laws of addition, • scalar multiplication. iv) Represent a vector in a Cartesian plane by defining fundamental unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$. v) Recognize all above definitions using analytical representation. vi) Find a unit vector in the direction of another given vector. vii) Find the position vector of a point which divides the line segment joining two points in a given ratio. viii) Use vectors to prove simple theorems of descriptive geometry.
3.2 Vectors in Space	i) Recognize rectangular coordinate system in space. ii) Define unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. iii) Recognize components of a vector. iv) Give analytic representation of a vector. v) Find magnitude of a vector. vi) Repeat all fundamental definitions for vectors in space which, in the plane, have already been discussed.
3.3 Properties of Vector Addition	i) State and prove • commutative law for vector addition. • associative law for vector addition. ii) Prove that: • $\mathbf{0}$ as the identity for vector addition. • $-\boldsymbol{A}$ as the inverse for $\boldsymbol{A}$.

3.4 Properties of Scalar Multiplication	State and prove: • commutative law for scalar multiplication, • associative law for scalar multiplication, • distributive laws for scalar multiplication.
3.5 Dot or Scalar Product	i) Define dot or scalar product of two vectors and give its geometrical interpretation. ii) Prove that: • $\boldsymbol{i}\cdot\boldsymbol{i}=\boldsymbol{j}\cdot\boldsymbol{j}=\boldsymbol{k}\cdot\boldsymbol{k}=1$, • $\boldsymbol{i}\cdot\boldsymbol{j}=\boldsymbol{j}\cdot\boldsymbol{k}=\boldsymbol{k}\cdot\boldsymbol{i}=0$. iii) Express dot product in terms of components. iv) Find the condition for orthogonality of two vectors. v) Prove the commutative and distributive laws for dot product. vi) Explain direction cosines and direction ratios of a vector. vii) Prove that the sum of the squares of direction cosines is unity. viii) Use dot product to find the angle between two vectors. ix) Find the projection of a vector along another vector. x) Find the work done by a constant force in moving an object along a given vector.
3.6 Cross or Vector Product	i) Define cross or vector product of two vectors and give its geometrical interpretation. ii) Prove that: • $\boldsymbol{i}\times\boldsymbol{i}=\boldsymbol{j}\times\boldsymbol{j}=\boldsymbol{k}\times\boldsymbol{k}=\boldsymbol{0}$, • $\boldsymbol{i}\times\boldsymbol{j}=-\boldsymbol{j}\times\boldsymbol{i}=\boldsymbol{k}$, • $\boldsymbol{j}\times\boldsymbol{k}=-\boldsymbol{k}\times\boldsymbol{j}=\boldsymbol{i}$, • $\boldsymbol{k}\times\boldsymbol{i}=-\boldsymbol{i}\times\boldsymbol{k}=\boldsymbol{j}$.

	iii) Express cross product in terms of components. iv) Prove that the magnitude of $\boldsymbol{A}\times\boldsymbol{B}$ represents the area of a parallelogram with adjacent sides $\boldsymbol{A}$ and $\boldsymbol{B}$. v) Find the condition for parallelism of two non-zero vectors. vi) Prove that $\boldsymbol{A}\times\boldsymbol{B}=-\boldsymbol{B}\times\boldsymbol{A}$. vii) Prove the distributive laws for cross product. viii) Use cross product to find the angle between two vectors. ix) Find the vector moment of a given force about a given point.
3.7 Scalar Triple Product	i) Define scalar triple product of vectors. ii) Express scalar triple product of vectors in terms of components (determinantal form). iii) Prove that: • $\boldsymbol{i}\cdot\boldsymbol{j}\times\boldsymbol{k}=\boldsymbol{j}\cdot\boldsymbol{k}\times\boldsymbol{i}=\boldsymbol{k}\cdot\boldsymbol{i}\times\boldsymbol{j}=1$, • $\boldsymbol{i}\cdot\boldsymbol{k}\times\boldsymbol{j}=\boldsymbol{j}\cdot\boldsymbol{i}\times\boldsymbol{k}=\boldsymbol{k}\cdot\boldsymbol{j}\times\boldsymbol{i}=-1$. iv) Prove that dot and cross are inter-changeable in scalar triple product. v) Find the volume of • a parallelepiped, • a tetrahedron, determined by three given vectors. vi) Define coplanar vectors and find the condition for coplanarity of three vectors.

UNIT 4 SEQUENCES AND SERIES

4.1 Sequence	i) Define a sequence (progression) and its terms. ii) Know that a sequence can be constructed from a formula or an inductive definition. iii) Recognize triangle, factorial and Pascal sequences.

4.2 Arithmetic Sequence	i)	Define an arithmetic sequence.
	ii)	Find the n th or general term of an arithmetic sequence.
	iii)	Solve problems involving arithmetic sequence.
4.3 Arithmetic Mean	i)	Know arithmetic mean between two numbers.
	ii)	Insert n arithmetic means between two numbers.
4.4 Arithmetic Series	i)	Define an arithmetic series.
	ii)	Establish the formula to find the sum to n terms of an arithmetic series.
	iii)	Show that sum of n arithmetic means between two numbers is equal to n times their arithmetic mean.
	iv)	Solve real life problems involving arithmetic series.
4.5 Geometric Sequence	i)	Define a geometric sequence.
	ii)	Find the n th or general term of a geometric sequence.
	iii)	Solve problems involving geometric sequence.
4.6 Geometric Mean	i)	Know geometric mean between two numbers.
	ii)	Insert n geometric means between two numbers.
4.7 Geometric Series	i)	Define a geometric series.
	ii)	Find the sum of n terms of a geometric series.
	iii)	Find the sum of an infinite geometric series.
	iv)	Convert the recurring decimal into an equivalent common fraction.
	v)	Solve real life problems involving geometric series.
4.8 Harmonic Sequence	i)	Recognize a harmonic sequence.
	ii)	Find nth term of harmonic sequence.
4.9 Harmonic Mean	i)	Define a harmonic mean.
	ii)	Insert n harmonic means between two numbers.

UNIT 5 MISCELLANEOUS SERIES

5.1 Evaluation of $\sum n, \sum n^2$ and $\sum n^3$	i) ii)	Recognize sigma ($\sum$) notation. Find sum of • the first n natural numbers ($\sum n$), • the squares of the first n natural numbers ($\sum n^2$), • the cubes of the first n natural numbers ($\sum n^3$).
5.2 Arithmetico-Geometric Series	i) ii)	Define arithmetico-geometric series. Find sum to n terms of the arithmetico-geometric series.
5.3 Method of Differences		Define method of differences. Use this method to find the sum of n terms of the series whose differences of the consecutive terms are either in arithmetic or in geometric sequence.
5.4 Summation of Series using Partial Fractions		Use partial fractions to find the sum to n terms and to infinity the series of the type $\frac{1}{a(a+d)}+\frac{1}{(a+d)(a+2d)}+\cdots$

UNIT 6 PERMUTATION, COMBINATION AND PROBABILITY

6.1 Factorial of a Natural Number		Know Kramp's factorial notation to express the product of first n natural numbers by $n!$.
6.2 Permutation	i) ii) iii)	Recognize the fundamental principle of counting and illustrate this principle using tree diagram. Explain the meaning of permutation of n different objects taken r at a time and know the notation ${}^nP_r$. Prove that ${}^nP_r = n(n-1)(n-2)\cdots(n-r+1)$ and hence deduce that • ${}^nP_r = \frac{n!}{(n-r)!}$, • ${}^nP_n = n!$, • 0!=1.

	iv)	Apply ${}^nP_r$ to solve relevant problems of finding the number of arrangements of n objects taken r at a time (when all n objects are different and when some of them are alike).
	v)	Find the arrangement of different objects around a circle.
6.3 Combination	i)	Define combination of n different objects taken r at a time.
	ii)	Prove the formula ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ and deduce that • $\binom{n}{n} = \binom{n}{0} = 1$, • $\binom{n}{r} = \binom{n}{n-r}$, $\binom{n}{1} = \binom{n}{n-1} = n$, • $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$.
	iii)	Solve problems involving combination.
6.4 Probability	i)	Define the following: • statistical experiment, • sample space and an event, • mutually exclusive events, • equally likely events, • dependent and independent events, • simple and compound events.
	ii)	Recognize the formula for probability of occurrence of an event E, that is $P(E) = \frac{n(E)}{n(S)}$, $0 \le P(E) \le 1$.

	iii)	Apply the formula for finding probability in simple cases.
	iv)	Use Venn diagrams and tree diagrams to find the probability for the occurrence of an event.
	v)	Define the conditional probability.
	vi)	Recognize the addition theorem (or law) of probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where A and B are two events. Deduce that $P(A \cup B) = P(A) + P(B)$ where A and B are mutually exclusive events.
	vii)	Recognize the multiplication theorem (or law) of probability $P(A \cap B) = P(A)P(B\|A)$ or $P(A \cap B) = P(B)P(A\|B)$ where $P(B\|A)$ and $P(A\|B)$ are conditional probabilities. Deduce that $P(A \cap B) = P(A)P(B)$ where A and B are independent events.
	viii)	Use theorems of addition and multiplication of probability to solve related problems.

UNIT 7 MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

7.1 Mathematical Induction	i)	Describe principle of mathematical induction.
	ii)	Apply the principle to prove the statements, identities or formulae.
7.2 Binomial Theorem	i)	Use Pascal's triangle to find the expansion of $(x+y)^n$ where n is a small positive integer.
	ii)	State and prove binomial theorem for positive integral index.

	iii)	Expand $(x+y)^n$ using binomial theorem and find its general term.
	iv)	Find the specified term in the expansion of $(x+y)^n$.
7.3 Binomial Series	i)	Expand $(1+x)^n$ where *n* is a positive integer and extend this result for all rational values of *n.*
	ii)	Expand $(1+x)^n$ in ascending powers of *x* and explain its validity/convergence for $\|x\|<1$ where *n* is a rational number.
	iii)	Determine the approximate values of the binomial expansions having indices as –ve integers or fractions.

UNIT 8 FUNCTIONS AND GRAPHS

8.1 Function	i)	Recall • function as a rule of correspondence, • domain, co-domain and range of a function, • one to one and onto functions.
	ii)	Know linear, quadratic and square root functions.
8.2 Inverse Function		Define inverse functions and demonstrate their domain and range with examples.
8.3 Graphical Representation of Functions	i)	Sketch graphs of • linear functions (e.g. $y=ax+b$), • non-linear functions (e.g. $y=x^2$).
	ii)	Sketch the graph of the function $y=x^n$ where n is • a + ve integer, • a – ve integer ($x\neq 0$), • a rational number for $x>0$.

	iii)	Sketch graph of quadratic function of the form $y = ax^2 + bx + c$, $a(\neq 0)$, b, c are integers.
	iv)	Sketch graph using factors.
	v)	Predict functions from their graphs (use the factor form to predict the equation of a function of the type $f(x) = ax^2 + bx + c$, if two points where the graph crosses x-axis and third point on the curve, are given).
8.4 Intersecting Graphs	i)	Find the intersecting point graphically when intersection occurs between • a linear function and coordinate axes, • two linear functions, • a linear and a quadratic function.
	ii)	Solve, graphically, appropriate problems from daily life.

UNIT 9 LINEAR PROGRAMMING

9.1 Introduction		Define linear programming (LP) as planning of allocation of limited resources to obtain an optimal result.
9.2 Linear Inequalities	i)	Find algebraic solutions of linear inequalities in one variable and represent them on number line.
	ii)	Interpret graphically the linear inequalities in two variables.
	iii)	Determine graphically the region bounded by up to 3 simultaneous linear inequalities of non-negative variables and shade the region bounded by them.
9.3 Feasible Region	i)	Define • linear programming problem, • objective function,

	• problem constraints, • decision variables. ii) Define and show graphically the feasible region (or solution space) of an LP problem. iii) Identify the feasible region of simple LP problems.
9.4 Optimal Solution	i) Define optimal solution of an LP problem. ii) Find optimal solution (graphical) through the following systematic procedure: • establish the mathematical formulation of LP problem, • construct the graph, • identify the feasible region, • locate the solution points, • evaluate the objective function, • select the optimal solution, • verify the optimal solution by actually substituting values of variables from the feasible region. iii) Solve real life simple LP problems.

UNIT 10 TRIGONOMETRIC IDENTITIES OF SUM AND DIFFERENCE OF ANGLES

10.1 Fundamental Law of Trigonometry	Use distance formula to establish fundamental law of trigonometry: • $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$, and deduce that • $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$, • $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$,

	• $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$.
10.2 Trigonometric Ratios of Allied Angles	i) Define allied angles. ii) Use fundamental law and its deductions to derive trigonometric ratios of allied angles. iii) Express $a\sin\theta + b\cos\theta$ in the form $r\sin(\theta + \phi)$ where $a = r\cos\phi$ and $b = r\sin\phi$.
10.3 Double, Half and Triple Angle Identities	Derive double angle, half angle and triple angle identities from fundamental law and its deductions.
10.4 Sum, Difference and Product of sine and cosine	i) Express the product (of sines and cosines) as sums or differences (of sines and cosines). ii) Express the sums or differences (of sines and cosines) as products (of sines and cosines).

UNIT 11 APPLICATION OF TRIGONOMETRY

11.1 Solving Triangles	i) Solve right angled triangle when measures of • two sides are given, • one side and one angle are given. ii) Define an oblique triangle and prove • the law of cosines, • the law of sines, • the law of tangents, and deduce respective half angle formulae. iii) Apply above laws to solve oblique triangles.
11.2 Area of a Triangle	Derive the formulae to find the area of a triangle in terms of the measures of • two sides and their included angle, • one side and two angles,

	• three sides (Hero's formula)
11.3 Circles Connected with Triangle	i) Define circum-circle, in-circle and escribed-circle. ii) Derive the formulae to find • circum-radius, • in-radius, • escribed-radii, and apply them to deduce different identities.

UNIT 12 GRAPHS OF TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS AND SOLUTION OF TRIGONOMETRIC EQUATIONS

12.1 Period of Trigonometric Functions	i) Find the domain and range of the trigonometric functions. ii) Define even and odd functions. iii) Discuss the periodicity of trigonometric functions. iv) Find the maximum and minimum value of a given function of the type: • $a+b\sin\theta$, • $a+b\cos\theta$, • $a+b\sin(c\theta+d)$, • $a+b\cos(c\theta+d)$, • the reciprocals of above, where *a, b, c* and *d* are real numbers.
12.2 Graphs of Trigonometric Functions	i) Recognize the shapes of the graphs of sine, cosine and tangent for all angles. ii) Draw the graphs of the six basic trigonometric functions within the domain from -2π to 2π. iii) Guess the graphs of $\sin 2\theta$, $\cos 2\theta$, $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$ etc. without actually drawing them. iv) Define periodic, even/odd and translation properties of the graphs of $\sin\theta$, $\cos\theta$ and $\tan\theta$, i.e., $\sin\theta$

		has: • periodic property $\sin(\theta \pm 2\pi) = \sin\theta$, • odd property $\sin(-\theta) = -\sin\theta$, • translation property $\begin{cases}\sin(\theta-\pi) = -\sin\theta, \\ \sin(\pi-\theta) = \sin\theta.\end{cases}$
	v)	Deduce $\sin(\theta + 2k\pi) = \sin\theta$ where k is an integer.
12.3 Solving Trigonometric Equations Graphically	i)	Solve trigonometric equations of the type $\sin\theta = k$, $\cos\theta = k$ and $\tan\theta = k$, using periodic, even/odd and translation properties.
	ii)	Solve graphically the trigonometric equations of the type: • $\sin\theta = \frac{\theta}{2}$, • $\cos\theta = \theta$, • $\tan\theta = 2\theta$ when $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.
12.4 Inverse Trigonometric Functions	i)	Define the inverse trigonometric functions and their domain and range.
	ii)	Find domains and ranges of • principal trigonometric functions, • inverse trigonometric functions.
	iii)	Draw the graphs of inverse trigonometric functions.
	iv)	Prove the addition and subtraction formulae of inverse trigonometric functions.
	v)	Apply addition and subtraction formulae of inverse trigonometric functions to verify related identities.
12.5 Solving General Trigonometric Equations	i)	Solve trigonometric equations and check their roots by substitution in the given trigonometric equations so as to discard extraneous roots.
	ii)	Use the periods of trigonometric functions to find the solution of general trigonometric equations.

CURRICULUM FOR MATHEMATICS – GRADE XII

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1 INTRODUCTION TO SYMBOLIC PACKAGE: MAPLE

1.1 Introduction	i) Recognize MAPLE environment. ii) Recognize basic MAPLE commands. iii) Use MAPLE as a calculator. iv) Use online MAPLE help.
1.2 Polynomials	Use MAPLE commands for • factoring a polynomial, • expanding an expression, • simplifying an expression, • simplifying a rational expression, • substituting into an expression.
1.3 Graphics	i) Plot a two-dimensional graph. ii) Demonstrate domain and range of a plot. iii) Sketch parametric equations. iv) Know plotting options.
1.4 Matrices	i) Recognize matrix and vector entry arrangement. ii) Apply matrix operations. iii) Compute inverse and transpose of a matrix.

UNIT 2 FUNCTIONS AND LIMITS

2.1 Functions	i) Identify through graph the domain and range of a function. ii) Draw the graph of modulus function (i.e. $y = \|x\|$) and identify its domain and range.

2.2 Composition of Functions	i) ii)	Recognize the composition of functions. Find the composition of two given functions.
2.3 Inverse of Composition of Functions		Describe the inverse of composition of two given functions.
2.4 Transcendental Functions		Recognize algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic (and their identities), explicit and implicit functions, and parametric representation of functions.
2.5 Graphical Representations	i)	Display graphically: • the explicitly defined functions like $y = f(x)$, where $f(x) = e^x, a^x, \log_a x, \log_e x$. • the implicitly defined functions such as $x^2 + y^2 = a^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and distinguish between graph of a function and of an equation. • the parametric equations of functions such as $x = at^2$, $y = 2at$; $x = a\sec\theta, y = b\tan\theta$. • the discontinuous functions of the type $y = \begin{cases} x & \text{when } 0 \le x < 1, \\ x-1 & \text{when } 1 \le x \le 2. \end{cases}$
	ii)	Use MAPLE graphic commands for two-dimensional plot of: • an expression (or a function), • parameterized form of a function, • implicit function, by restricting domain and range.
	iii)	Use MAPLE package plots for plotting different types of functions.

2.6 Limit of a Function	i)	Identify a real number by a point on the number line.
	ii)	Define and represent • open interval, • closed interval, • half open and half closed intervals, on the number line.
	iii)	Explain the meaning of phrase: • x tends to zero ($x \to 0$), • x tends to a ($x \to a$), • x tends to infinity ($x \to \infty$).
	iv)	Define limit of a sequence.
	v)	Find the limit of a sequence whose nth term is given.
	vi)	Define limit of a function.
	vii)	State the theorems on limits of sum, difference, product and quotient of functions and demonstrate through examples.
2.7 Important Limits	i)	Evaluate the limits of functions of the following types: • $\frac{x^n - a^n}{x - a}$, $\frac{x - a}{\sqrt{x} - \sqrt{a}}$ when $x \to a$, • $\left(1 + \frac{1}{x}\right)^x$ when $x \to \infty$, • $(1+x)^{\frac{1}{x}}$, $\frac{\sqrt{x+a} - \sqrt{a}}{x}$, $\frac{a^x - 1}{x}$, $\frac{(1+x)^n - 1}{x}$, and $\frac{\sin x}{x}$ when $x \to 0$.
	ii)	Evaluate limits of different algebraic, exponential and trigonometric functions.
	iii)	Use MAPLE command limit to evaluate limit of a function.

2.8 Continuous and Discontinuous Functions	i)	Recognize left hand and right hand limits and demonstrate through examples.
	ii)	Define continuity of a function at a point and in an interval.
	iii)	Test continuity and discontinuity of a function at a point and in an interval.
	iv)	Use MAPLE command **iscont** to test continuity of a function at a point and in a given interval.

UNIT 3 DIFFERENTIATION

3.1 Derivative of a Function	i)	Distinguish between independent and dependent variables.
	ii)	Estimate corresponding change in the dependent variable when independent variable is incremented (or decremented).
	iii)	Explain the concept of a rate of change.
	iv)	Define derivative of a function as an instantaneous rate of change of a variable with respect to another variable.
	v)	Define derivative or differential coefficient of a function.
	vi)	Differentiate $y = x^n$, where $n \in Z$ (the set of integers), from first principles (the derivation of power rule).
	vii)	Differentiate $y = (ax+b)^n$, where $n = \frac{p}{q}$ and p, q are integers such that $q \neq 0$, from first principles.
3.2 Theorems on Differentiation		Prove the following theorems for differentiation. • The derivative of a constant is zero. • The derivative of any constant multiple of a

	function is equal to the product of that constant and the derivative of the function. • The derivative of a sum (or difference) of two functions is equal to the sum (or difference) of their derivatives. • The derivative of a product of two functions is equal to (the first function)×(derivative of the second function) plus (derivative of the first function)×(the second function). • The derivative of a quotient of two functions is equal to denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.
3.3 Application of Theorems on Differentiation	Differentiate: • constant multiple of x^n, • sum (or difference) of functions, • polynomials, • product of functions, • quotient of two functions.
3.4 Chain Rule	i) Prove that $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ when $y=f(u)$ and $u=g(x)$. ii) Show that $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$. iii) Use chain rule to show that $\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}f'(x)$. iv) Find derivative of implicit function.

3.5 Differentiation of Trigonometric and Inverse Trigonometric Functions		Differentiate: • trigonometric functions ($\sin x$, $\cos x$, $\tan x$, $\text{cosec}x$, $\sec x$, and $\cot x$) from first principles. • inverse trigonometric functions ($\arcsin x$, $\arccos x$, $\arctan x$, $\text{arccosec}x$, $\text{arcsec}x$, and $\text{arccot}x$) using differentiation formulae.
3.6 Differentiation of Exponential and Logarithmic Functions	i) ii) iii)	Find the derivative of e^x and a^x from first principles. Find the derivative of $\ln x$ and $\log_a x$ from first principles. Use logarithmic differentiation to find derivative of algebraic expressions involving product, quotient and power.
3.7 Differentiation of Hyperbolic and Inverse Hyperbolic Functions		Differentiate: • hyperbolic functions ($\sinh x$, $\cosh x$, $\tanh x$, $\text{cosech}\, x$, $\text{sech}\, x$ and $\coth x$). • inverse hyperbolic functions ($\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$, $\text{cosech}^{-1} x$, $\text{sech}^{-1} x$, and $\coth^{-1} x$).
		Use MAPLE command **diff** to differentiate a function.

UNIT 4 HIGHER ORDER DERIVATIVES AND APPLICATIONS

4.1 Higher Order Derivatives	i) ii) iii)	Find higher order derivatives of algebraic, trigonometric, exponential and logarithmic functions. Find the second derivative of implicit, inverse trigonometric and parametric functions. Use MAPLE command **diff** repeatedly to find higher order derivative of a function.

4.2 Maclaurin's and Taylor's Expansions	i)	State Maclaurin's and Taylor's theorems (without remainder terms). Use these theorems to expand $\sin x, \cos x, \tan x$, a^x, e^x, $\log_a(1+x)$ and $\ln(1+x)$.
	ii)	Use MAPLE command **taylor** to find Taylor's expansion for a given function.
4.3 Application of Derivatives	i)	Give geometrical interpretation of derivative.
	ii)	Find the equation of tangent and normal to the curve at a given point.
	iii)	Find the angle of intersection of the two curves.
	iv)	Find the point on a curve where the tangent is parallel to the given line.
4.4 Maxima and Minima	i)	Define increasing and decreasing functions.
	ii)	Prove that if $f(x)$ is a differentiable function on the open interval (a,b) then • $f(x)$ is increasing on (a,b) if $f'(x) > 0, \forall x \in (a,b)$, • $f(x)$ is decreasing on (a,b) if $f'(x) < 0, \forall x \in (a,b)$.
	iii)	Examine a given function for extreme values.
	iv)	State the second derivative rule to find the extreme values of a function at a point.
	v)	Use second derivative rule to examine a given function for extreme values.
	vi)	Solve real life problems related to extreme values.
	vii)	Use MAPLE command **maximize (minimize)** to compute maximum (minimum) value of a function.

UNIT 5 DIFFERENTIATION OF VECTOR FUNCTIONS

5.1 Scalar and Vector Functions	i)	Define scalar and vector function.
	ii)	Explain domain and range of a vector function.
5.2 Limit and Continuity	i)	Define limit of a vector function and employ the usual technique for algebra of limits of scalar function to demonstrate the following properties of limits of a vector function. • The limit of the sum (difference) of two vector functions is the sum (difference) of their limits. • The limit of the dot product of two vector functions is the dot product of their limits. • The limit of the cross product of two vector functions is the cross product of their limits. • The limit of the product of a scalar function and a vector function is the product of their limits.
	ii)	Define continuity of a vector function and demonstrate through examples.
5.3 Derivative of Vector Function		Define derivative of a vector function of a single variable and elaborate the result: if $\boldsymbol{f}(t) = f_1(t)\boldsymbol{i} + f_2(t)\boldsymbol{j} + f_3(t)\boldsymbol{k}$, where $f_1(t), f_2(t), f_3(t)$ are differentiable functions of a scalar variable t, then $\frac{d\boldsymbol{f}}{dt} = \frac{df_1}{dt}\boldsymbol{i} + \frac{df_2}{dt}\boldsymbol{j} + \frac{df_3}{dt}\boldsymbol{k}$.
5.4 Vector Differentiation	i)	Prove the following formulae of differentiation: • $\frac{d\boldsymbol{a}}{dt} = \boldsymbol{0}$, • $\frac{d}{dt}[\boldsymbol{f} \pm \boldsymbol{g}] = \frac{d\boldsymbol{f}}{dt} \pm \frac{d\boldsymbol{g}}{dt}$, • $\frac{d}{dt}[\phi \boldsymbol{f}] = \phi\frac{d\boldsymbol{f}}{dt} + \frac{d\phi}{dt}\boldsymbol{f}$,

	• $\frac{d}{dt}[\boldsymbol{f}\cdot\boldsymbol{g}]=\boldsymbol{f}\cdot\frac{d\boldsymbol{g}}{dt}+\frac{d\boldsymbol{f}}{dt}\cdot\boldsymbol{g}$, • $\frac{d}{dt}[\boldsymbol{f}\times\boldsymbol{g}]=\boldsymbol{f}\times\frac{d\boldsymbol{g}}{dt}+\frac{d\boldsymbol{f}}{dt}\times\boldsymbol{g}$, • $\frac{d}{dt}\left[\frac{\boldsymbol{f}}{\phi}\right]=\frac{1}{\phi^2}\left[\phi\frac{d\boldsymbol{f}}{dt}-\frac{d\phi}{dt}\boldsymbol{f}\right]$, where $\boldsymbol{a}$ is a constant vector function, $\boldsymbol{f}$ and $\boldsymbol{g}$ are vector functions, and ϕ is a scalar function of t. ii) Apply vector differentiation to calculate velocity and acceleration of a position vector $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$.

UNIT 6 INTEGRATION

6.1 Introduction	i) Demonstrate the concept of the integral as an accumulator. ii) Know integration as inverse process of differentiation. iii) Explain constant of integration. iv) Know simple standard integrals which directly follow from standard differentiation formulae.
6.2 Rules of Integration	i) Recognize the following rules of integration. • $\int\frac{d}{dx}[f(x)]dx=\frac{d}{dx}[\int f(x)dx]=f(x)+c$, • where c is a constant of integration. • The integral of the product of a constant and a function is the product of the constant and the integral of the function. • The integral of the sum of a finite number of functions is equal to the sum of their integrals. ii) Use standard differentiation formulae to prove the results for the following integrals:

	• $\int [f(x)]^n f'(x)dx$, • $\int \frac{f'(x)}{f(x)}dx$, • $\int e^{ax}[af(x)+f'(x)]dx$.
6.3 Integration by Substitution	i) Explain the method of integration by substitution. ii) Apply method of substitution to evaluate indefinite integrals. iii) Apply method of substitution to evaluate integrals of the following types: • $\int \frac{dx}{a^2-x^2}$, $\int \sqrt{a^2-x^2}dx$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, • $\int \frac{dx}{a^2+x^2}$, $\int \sqrt{a^2+x^2}dx$, $\int \frac{dx}{\sqrt{x^2+a^2}}$, • $\int \frac{dx}{x^2-a^2}$, $\int \sqrt{x^2-a^2}dx$, $\int \frac{dx}{\sqrt{x^2-a^2}}$, • $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, • $\int \frac{px+q}{ax^2+bx+c}dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$.
6.4 Integration by Parts	i) Recognize the formula for integration by parts. ii) Apply method of integration by parts to evaluate integrals of the following types: • $\int \sqrt{a^2-x^2}dx$, $\int \sqrt{a^2+x^2}dx$, $\int \sqrt{x^2-a^2}dx$. iii) Evaluate integrals using integration by parts.
6.5 Integration using Partial Fractions	Use partial fractions to find $\int \frac{f(x)}{g(x)}dx$, where $f(x)$

		and $g(x)$ are algebraic functions such that $g(x) \neq 0$.
6.6 Definite Integrals	i)	Define definite integral as the limit of a sum.
	ii)	Describe the fundamental theorem of integral calculus and recognize the following basic properties: • $\int_a^a f(x)dx = 0$, • $\int_a^b f(x)dx = \int_a^b f(y)dy$, • $\int_a^b f(x)dx = -\int_b^a f(x)dx$, • $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,\ a < c < b$, • $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{when } f(-x) = f(x), \\ 0 & \text{when } f(-x) = -f(x). \end{cases}$
	iii)	Extend techniques of integration using properties to evaluate definite integrals.
	iv)	Represent definite integral as the area under the curve.
	v)	Apply definite integrals to calculate area under the curve.
	vi)	Use MAPLE command int to evaluate definite and indefinite integrals.

UNIT 7 PLANE ANALYTIC GEOMETRY – STRAIGHT LINE

7.1 Division of a Line Segment	i)	Recall distance formula to calculate distance between two points given in Cartesian plane.
	ii)	Find coordinates of a point that divides the line segment in given ratio (internally and externally).

	iii)	Show that the medians and angle bisectors of a triangle are concurrent.
7.2 Slope of a Straight Line	i) ii) iii)	Define the slope of a line. Derive the formula to find the slope of a line passing through two points. Find the condition that two straight lines with given slopes may be • parallel to each other, • perpendicular to each other.
7.3 Equation of a Straight Line Parallel to Co-ordinate Axes		Find the equation of a straight line parallel to • y-axis and at a distance a from it, • x-axis and at a distance b from it.
7.4 Standard Form of Equation of a Straight Line	i) ii) iii)	Define intercepts of a straight line. Derive equation of a straight line in • slope-intercept form, • point-slope form, • two-point form, • intercepts form, • symmetric form, • normal form. Show that a linear equation in two variables represents a straight line. Reduce the general form of the equation of a straight line to the other standard forms.
7.5 Distance of a Point From a Line	i) ii)	Recognize a point with respect to position of a line. Find the perpendicular distance from a point to the given straight line.
7.6 Angle Between Lines	i)	Find the angle between two coplanar intersecting straight lines.

	ii)	Find the equation of family of lines passing through the point of intersection of two given lines.
	iii)	Calculate angles of the triangle when the slopes of the sides are given.
7.7 Concurrency of Straight Lines	i)	Find the condition of concurrency of three straight lines.
	ii)	Find the equation of median, altitude and right bisector of a triangle.
	iii)	Show that • three right bisectors, • three medians, • three altitudes, of a triangle are concurrent.
7.8 Area of a Triangular Region		Find area of a triangular region whose vertices are given.
7.9 Homogenous Equation	i)	Recognize homogeneous linear and quadratic equations in two variables.
	ii)	Investigate that the 2nd degree homogeneous equation in two variables x and y represents a pair of straight lines through the origin and find acute angle between them.

UNIT 8 CONICS – I

8.1 Introduction		Define conics and demonstrate members of its family i.e. circle, parabola, ellipse and hyperbola.
8.2 Circle		
8.2.1 Equation of a Circle	i)	Define circle and derive its equation in standard form i.e. $(x-h)^2+(y-k)^2=r^2$.
8.2.2 General Form of an	ii)	Recognize general equation of a circle

Equation of a Circle 8.2.3 Equation of Circle determined by a given condition		$x^2 + y^2 + 2gx + 2fy + c = 0$ and find its centre and radius. iii) Find the equation of a circle passing through • three non-collinear points, • two points and having its centre on a given line, • two points and equation of tangent at one of these points is known, • two points and touching a given line.
8.3 Tangent and Normal	i) ii) iii) iv) v) vi)	Find the condition when a line intersects the circle. Find the condition when a line touches the circle. Find the equation of a tangent to a circle in slope form. Find the equations of a tangent and a normal to a circle at a point. Find the length of tangent to a circle from a given external point. Prove that two tangents drawn to a circle from an external point are equal in length.
8.4 Properties of Circle		Prove analytically the following properties of a circle. • Perpendicular from the centre of a circle on a chord bisects the chord. • Perpendicular bisector of any chord of a circle passes through the centre of the circle. • Line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord. • Congruent chords of a circle are equidistant from its centre and its converse. • Measure of the central angle of a minor arc is

	double the measure of the angle subtended by the corresponding major arc. • An angle in a semi-circle is a right angle. • The perpendicular at the outer end of a radial segment is tangent to the circle. • The tangent to a circle at any point of the circle is perpendicular to the radial segment at that point.

UNIT 9 CONICS – II

9.1 Parabola	i)	Define parabola and its elements (i.e. focus, directrix, eccentricity, vertex, axis, focal chord and latus rectum).
9.1.1 General Form of Equation of a Parabola	ii)	Derive the general form of an equation of a parabola.
9.1.2 Standard Form of Equation of Parabola	iii)	Derive the standard equations of parabola, sketch their graphs and find their elements.
	iv)	Find the equation of a parabola with the following given elements: • focus and vertex, • focus and directrix, • vertex and directrix.
9.1.3 Equations of Tangent and Normal	v)	Recognize tangent and normal to a parabola.
	vi)	Find the condition when a line is tangent to a parabola at a point and hence write the equation of a tangent line in slope form.
	vii)	Find the equation of a tangent and a normal to a parabola at a point.
9.1.4 Application of Parabola	viii)	Solve suspension and reflection problems related to parabola.

9.2 Ellipse	i)	Define ellipse and its elements (i.e. centre, foci, vertices, covertices, directrices, major and minor axes, eccentricity, focal chord and latera recta).
	ii)	Explain that circle is a special case of an ellipse.
9.2.1 Standard Form of Equation of an Ellipse	iii)	Derive the standard form of equation of an ellipse and identify its elements.
	iv)	Find the equation of an ellipse with the following given elements • major and minor axes, • two points, • foci, vertices or lengths of a latera recta, • foci, minor axes or length of a latus rectum.
	v)	Convert a given equation to the standard form of equation of an ellipse, find its elements and draw the graph.
9.2.2 Equations of Tangent and Normal	vi)	Recognize tangent and normal to an ellipse.
	vii)	Find points of intersection of an ellipse with a line including the condition of tangency.
	viii)	Find the equation of a tangent in slope form.
	ix)	Find the equation of a tangent and a normal to an ellipse at a point.
9.3 Hyperbola	i)	Define hyperbola and its elements (i.e. centre, foci, vertices, directrices, transverse and conjugate axes, eccentricity, focal chord and latera recta).
9.3.1 Standard Form of Equation of Hyperbola	ii)	Derive the standard form of equation of a hyperbola and identify its elements.
	iii)	Find the equation of a hyperbola with the following given elements: • transverse and conjugate axes with centre at origin,

		• two points, • eccentricity, latera recta and transverse axes, • focus, eccentricity and centre, • focus, centre and directrix.
9.3.2 Equation of Tangent and Normal	iv) v) vi) vii)	Convert a given equation to the standard form of equation of a hyperbola, find its elements and sketch the graph. Recognize tangent and normal to a hyperbola. Find • points of intersection of a hyperbola with a line including the condition of tangency, • the equation of a tangent in slope form. Find the equation of a tangent and a normal to a hyperbola at a point.
9.4 Translation and Rotation of Axes	i) ii) iii) iv) v)	Define translation and rotation of axes and demonstrate through examples. Find the equations of transformation for • translation of axes, • rotation of axes. Find the transformed equation by using translation or rotation of axes. Find new origin and new axes referred to old origin and old axes. Find the angle through which the axes be rotated about the origin so that the product term xy is removed from the transformed equation.

UNIT 10 DIFFERENTIAL EQUATIONS

10.1 Introduction	Define ordinary differential equation (DE), order of a

		DE, degree of a DE, solution of a DE – general solution and particular solution.
10.2 Formation of Differential Equations		Demonstrate the concept of formation of a differential equation.
10.3 Solution of Differential Equation	i)	Solve differential equations of first order and first degree of the form: • separable variables, • homogeneous equations, • equations reducible to homogeneous form.
	ii)	Solve real life problems related to differential equations.
10.4 Orthogonal Trajectories	i)	Find orthogonal trajectories (rectangular coordinates) of the given family of curves.
	ii)	Use MAPLE graphic commands to view the graphs of given family of curves and its orthogonal trajectories.

UNIT 11 PARTIAL DIFFERENTIATION

11.1 Differentiation of Function of Two Variables	i)	Define a function of two variables.
	ii)	Define partial derivative.
	iii)	Find partial derivatives of a function of two variables.
11.2 Euler's Theorem	i)	Define a homogeneous function of degree *n*.
	ii)	State and prove Euler's theorem on homogeneous functions.
	iii)	Verify Euler's theorem for homogeneous functions of different degrees (simple cases).
	iv)	Use MAPLE command **diff** to find partial derivatives.

UNIT 12 INTRODUCTION TO NUMERICAL METHODS

12.1 Numerical Solution of Non-linear Equations	i) Describe importance of numerical methods. ii) Explain the basic principles of solving a non-linear equation in one variable. iii) Calculate real roots of a non-linear equation in one variable by • bisection method, • regula-falsi method, • Newton-Raphson method. iv) Use MAPLE command fsolve to find numerical solution of an equation and demonstrate through examples.
12.2 Numerical Quadrature	i) Define numerical quadrature. Use • Trapezoidal rule, • Simpson's rule, to compute the approximate value of definite integrals without error terms. ii) Use MAPLE command trapezoid for trapezoidal rule and simpson for Simpson's rule and demonstrate through examples.

Teaching Strategies

Introduction

In the classrooms teachers transmit textbook facts to students, who in turn are expected to memorize and regurgitate them in examinations. The teachers are so ingrained that they find this method of lecture and recitation as a good way of teaching a large number of students in their classrooms. In Mathematics students memorize rules without understanding their rationale. There is no doubt that the timely reward to this way is more immediate and more apparent but this instrumental learning does not bring desired result subsequently. The memorized rules may work for a limited range of similar problems but students do not feel comfortable when they face different or challenging tasks. Consequently the students are totally dependent upon teachers. They cannot progress in thinking, hence their self-esteem is low.

To capture all aspects of expertise, competence, knowledge and facility which are necessary for anyone to learn Mathematics, Kilpatrick et al[†] (2001) present the notion of mathematical proficiency that is composed of following five interwoven but interdependent strands:

- **Conceptual understanding** – comprehension of mathematical concepts, operations and relations.
- **Procedural fluency** – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- **Strategic competence** – ability to formulate, represent and solve mathematical problems.
- **Adaptive reasoning** – capacity for logical thought, reflection, explanation and justification.
- **Productive disposition** – habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

Part I Teaching Mathematics – Role of a Teacher

Research indicates that teachers who have a good background in Mathematics also add richness to their lessons, involve students extensively in mathematical dialogue and capitalize on students' questions/discussions to weave/extend mathematical relationships. They do not list only the definitions and step-by-step procedures for students to memorize without understanding their meaning and function.

Teachers need to assume a new role if students are to construct their own mathematical understanding. Rather than just pouring mathematical knowledge into students' heads, teachers must create a stimulating environment that encourages mathematical learning through increasing interactivity.

Teaching in a Mathematics classroom requires listening to the students, understanding their level of thinking, setting a task and analysing outcomes of the task in order to understand how students construct meanings – this is contrary to a traditional way of teaching. The teachers' role shifts from dispensing information to planning investigative tasks, managing a cooperative learning environment and supporting students' creativity in developing rational understanding of the concepts. This improved teaching practice should include the following aspects of a teacher's role. The teacher should be

- a planner of practical tasks for the students to consolidate and organise their informal knowledge.

[†] Kilpatrick, J., Swafford, J., and Findell, B. (Eds) (2001) *Adding It Up: Helping Children Learn Mathematics*, Mathematics
[†] Learning Study Committee, Centre for Education, Washington, DC: National Academies Press.

- an organizer of the establishment of mathematical tasks in the classroom so that the students can work in a social setting and develop rational understanding.
- an encourager who asks questions, supports and develops students' mathematical thinking and communication.
- a negotiator helping students to discuss various meanings/solutions of a concept/question and to achieve a common agreement.
- a mediator supporting the establishment of an environment where students express opinions and experiences in the classroom equally.

Thus a teacher's primary responsibilities are to assist learners' cognitive reconstruction and conceptual re-organisation through providing them the opportunities for interaction in mathematical tasks that encourage discussion and negotiation of ideas to help them to develop conceptual understanding.

Part II Effective Teaching Strategies

Students learn things in many different ways. They do not always learn best by sitting and listening to the teacher. Students particularly of the primary level can learn by presentation and explanation by the teacher, consolidation and practice, games, practical work, problems and puzzles, and investigating Mathematics.

Investigating Mathematics

Teachers may set students a challenge, matched to their ability, which leads them to discover and practice some new Mathematics for themselves. The key point about investigations is that students are encouraged to make their own decisions about:

- where to start,
- how to deal with challenges,
- what Mathematics they need to use,
- how they can communicate this Mathematics,
- how to describe what they have discovered.

Problem Solving

A problem is a statement or proposition requiring an algebraic, geometric, or other mathematical solution. A widespread opinion that problem solving should be the central focus of the curriculum for Mathematics strengthens the fact that 'learning to solve problems is the principal reason for studying Mathematics'.

A problem exists when there is a situation a learner wants to resolve but no solution is readily apparent. For example Shahzaib counted 19 cycle-wheels, run by 7 cycle-riders, going past his house. How many tricycles were there? Working on this problem offers a good practice in addition, multiplication and division skills. But the important goal of this problem is to help students think systematically about possibilities and record thinking.

Research suggests that a problem solver needs to become better acquainted with a problem and works for a clearer understanding of it before progressing towards a solution. The path from understanding the problem to devising a plan may sometimes be long but experience and practice are the best teachers to contrive. The plan gives a general outline of direction to solve the problem. Having arrived at a result, it is verified by referring back to the original problem.

Part III Time Distribution

Teaching schedules are among the integral parts of Mathematics classrooms. They help school management to run and monitor the teaching of a particular subject. The following tables, indicating unit-wise time distribution, will be supportive to the teachers and education planners.

UNITWISE TIME DISTRIBUTION – GRADE VI

Unit	Title	Periods (40 minutes each)
1.	Sets	12
2.	Whole Numbers	12
3.	Factors and Multiples	30
4.	Integers	18
5.	Simplifications	18
6.	Ratio and Proportion	12
7.	Financial Arithmetic	12
8.	Introduction to Algebra	12
9.	Linear Equations	12
10.	Geometry	30
11.	Perimeter and Area	14
12.	Three Dimensional Solids	16
13.	Information Handling	12
	TOTAL	210 (6 periods per week)

UNITWISE TIME DISTRIBUTION – GRADE VII

Unit	Title	Periods (40 minutes each)
1.	Sets	14
2.	Rational Numbers	12
3.	Decimals	12
4.	Exponents	12
5.	Square Root of Positive Number	16
6.	Direct and Inverse Variation	16
7.	Financial Arithmetic	14
8.	Algebraic Expressions	26
9.	Linear Equations	10
10.	Fundamentals of Geometry	26
11.	Practical Geometry	26
12.	Circumference, Area and Volume	16
13.	Information Handling	10
	TOTAL	210 (6 periods per week)

UNITWISE TIME DISTRIBUTION – GRADE VIII

Unit	Title	Periods (40 minutes each)
1.	Operations on Sets	13
2.	Real Numbers	18
3.	Number Systems	10
4.	Financial Arithmetic	24
5.	Polynomials	13
6.	Factorization, Simultaneous Equations	33
7.	Fundamentals of Geometry	18
8.	Practical Geometry	24
9.	Areas and Volumes	14
10.	Demonstrative Geometry	16
11.	Introduction to Trigonometry	14
12.	Information Handling	13
	TOTAL	210 (6 periods per week)

UNITWISE TIME DISTRIBUTION – GRADES IX-X

Unit	Title	Periods (40 minutes each)
1.	Matrices and Determinants	18
2.	Real and Complex Numbers	14
3.	Logarithm	10
4.	Algebraic Expressions and Algebraic Formulas	16
5.	Factorization	16
6.	Algebraic Manipulation	14
7.	Linear Equations and Inequalities	12
8.	Quadratic equations	16
9.	Theory of Quadratic equations	28
10.	Variations	12
11.	Partial fractions	12
12.	Sets and Functions	26
13.	Basic Statistics	16
14.	Linear Graphs and their Applications	10
15.	Introduction to Coordinate Geometry	09
16.	Introduction to Trigonometry	23
17.	Congruent Triangles	12
18.	Parallelograms and Triangles	13
19.	Line Bisectors and Angle Bisectors	14
20.	Sides and Angles of a Triangle	10
21.	Ratio and Proportion	12
22.	Pythagoras Theorem	08
23.	Theorems related to Area	10
24.	Projection of a Side of a Triangle	14
25.	Chords of a Circle	13
26.	Tangent to a Circle	10
27.	Chords and Arcs	11
28.	Angle in a Segment of a Circle	14
29.	Practical Geometry – Triangles	12
30.	Practical Geometry – Circles	15
	TOTAL	420 (6 periods per week)

UNITWISE TIME DISTRIBUTION – GRADE XI

Unit	Title	Periods (40 minutes each)
1.	Complex Numbers	12
2.	Matrices and Determinants	48
3.	Vectors	24
4.	Sequences and Series	20
5.	Miscellaneous Series	08
6.	Permutation, Combination and Probability	24
7.	Mathematical Induction and Binomial Theorem	20
8.	Functions and Graphs	24
9.	Linear Programming	12
10.	Trigonometric Identities of Sum and Differences of Angles	24
11.	Application of Trigonometry	24
12.	Graphs of Trigonometric and Inverse Trigonometric Functions and Solution of Trigonometric Equations	40
	TOTAL	280 (8 periods per week)

UNITWISE TIME DISTRIBUTION – GRADE XII

Unit	Title	Periods (40 minutes each)
1.	Introduction to Symbolic Package: MAPLE	12
2.	Functions and Limits	32
3.	Differentiation	32
4.	Higher Order Derivatives and applications	16
5.	Differentiation of Vector Functions	12
6.	Integration	48
7.	Plane Analytic Geometry – Straight Line	32
8.	Conics-I	12
9.	Conics-II	32
10.	Differential Equations	18
11.	Partial Differentiation	10
12.	Introduction to Numerical Methods	24
	TOTAL	280 (8 periods per week)

Assessment and Evaluation

Introduction

Assessment is the process of gathering information using a variety of tools and techniques that reflect how well a student is achieving the curriculum expectations in a subject. As part of assessment teachers provide students with descriptive feedback that guides their efforts towards improvement. The quality of assessment largely determines the quality of evaluation. Evaluation refers to the process of judgments and decisions based on the interpretation of evidence gathered through assessment.

Rowntree[*] (1990) defined assessment as having two purposes: firstly to support and provide feedback to learners and improve their ongoing learning, and secondly to report on what they had already achieved. In essence the first is formative assessment and the second is summative assessment. Morgan and O'Reilly[†] (1999) believe that assessment is the engine that drives and shapes learning, rather than an end of course event that grades and reports on performance.

Assessment and evaluation should be based on curriculum expectations and the achievement levels outlined in the national curriculum. To ensure that assessment and evaluation lead to the improvement of student learning, teachers must use specific assessment and evaluation strategies that

- address both what students learn and how well they learn.
- are administered over a period of time and designed to provide opportunities for students to demonstrate full range of their learning.
- ensure that each student is given clear directions for improvement.
- promote students' ability to assess their own learning.
- are communicated clearly to students and parents in advance.

Part I Assessment in Mathematics

It should be kept in mind that in mathematics a single type of assessment can frustrate students, diminish their self-confidence and make them feel anxious about the subject. In reality the understanding of mathematical concepts encompasses a broad range of abilities. Examples of various templates to assess different abilities are mentioned below.

Assessment must include by focusing on a student's ability to:

- communicate mathematically.
- reason and analyze, and to think and act in positive ways.
- comprehend the key concepts.
- evaluate the effectiveness of using different strategies to address the same problem.
- use a variety of strategies to problem solving and to make mathematical connections.
- discriminate between relevant and irrelevant attributes of a concept in selecting examples.
- integrate and to make sense of mathematical concept and procedure.
- examine real life situations by reasoning mathematically.

[*] Rowntree, D. (1990) *Teaching through Self-Instruction (Second Ed)*, London: Kogan Page.

[†] Morgan, C. and O'Reilly, M. (1999) *Assessing Open and Distance Learners*, London: Kogan Page.

Learning of mathematics, being a cumulative process, occurs as experiences contribute to understanding. Suggested below are the assessment strategies to obtain valid and reliable picture of students' understanding and achievement.

i) **Classroom-based assessments** that include anecdotal records, checklists, rating scales, portfolios peer- and self-assessment.
ii) **Teacher-designed test formats** that include oral examination, assignments, short answers, matching, multiple-choice, fill-in and true-false items.

Part II The Traditional Examinations

Bearing in mind the requirement of simplicity in considering assessment strategies, the examinations in traditional paper-based mode with place and time-specific activities, are easy to organize for institutions (Boards of Intermediate and Secondary Education). When a formal examination, for Secondary School Certificate (SSC) or Higher Secondary School Certificate (HSSC), is used for assessment there are examination centres, infrastructure to supply and secure examination papers before examination and arrangements to check the identities of the candidates, invigilate the examination and collect the scripts for marking. Marks are then gathered and results are promulgated in a timely manner.

For assessment and evaluation of grade levels I-VIII the institutions adopt their own criteria. The means by which each institution achieves quality should differ according to the circumstances in which it operates, but each must give priority to meeting students' expectations in terms of learning outcomes they can legitimately expect to achieve. In essence an effective learning-outcomes-oriented quality assurance system must be based on constant monitoring and effective feedback loops.

Instructions for SSC and HSSC Examinations

The examining institutions or bodies including all Boards of Intermediate and Secondary Education for the conduct of SSC and HSSC examinations in the subject of Mathematics should follow instructions as given below.

(1) The question papers should be balanced in all respect. Following table, showing weightage to difficulty level of questions, is suggested to be a practicable criterion for a balanced question paper of Mathematics.

Difficulty Level of Questions	Weightage
Easy	15 %
Average	70 %
Difficult	15 %

(2) According to the new Scheme of Studies 150 marks have been allocated to the subject of Mathematics for SSC Examination. There will be two papers (Paper-A and Paper-B) of Mathematics each carrying 75 marks.

- For **Paper-A (Algebra)** the examiners will set the questions keeping in view the table (unitwise weightages for grades IX-X). The paper will cover units 1-13.
- For **Paper-B (Geometry)** the examiners will set the questions keeping in view the table (unitwise weightages for grades IX-X). The paper, with the following breakdown, will cover units 14-30.

Linear Graphs and their Application	5 %	20 %	100 %
Introduction to Coordinate Geometry	4 %		
Introduction to Trigonometry	11 %		
Descriptive Geometry	70 %	80 %	
Practical Geometry	10 %		

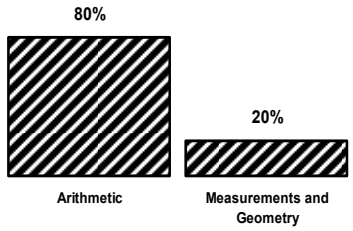
(3) For HSSC examination the examiners will set both the papers of Mathematics for Part-I and Part-II keeping in view the tables (unitwise weightages for grades XI and XII). To integrate technology the symbolic package MAPLE has been introduced first time in the curriculum of Mathematics for grade XII. In the question paper of Mathematics-Part II no question will be set from unit-I (Introduction to Symbolic Package: MAPLE) grade XII until the instructors are trained and institutions are equipped appropriately within next three academic years. Zero weightage has therefore been assumed for the said unit initially.

Part III Unitwise Weightages

Following tables explain weightages of specified topics with respect to different grade levels in accordance with the curriculum. Bar charts drawn alongside the tables hence reflect weightages of fundamental disciplines in the field of Mathematics.

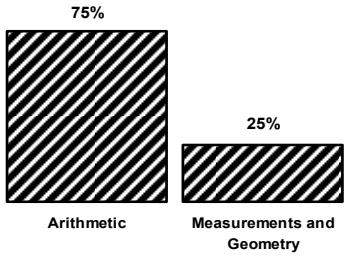
UNITWISE WEIGHTAGES – GRADE I

Unit	Title	Weightage
1.	Concept of Whole Numbers	30 %
2.	Number Operations	30 %
3.	Measurement of Length and Mass	10 %
4.	Money	10 %
5.	Time and Date	10 %
6.	Geometry	10 %
	TOTAL	100 %



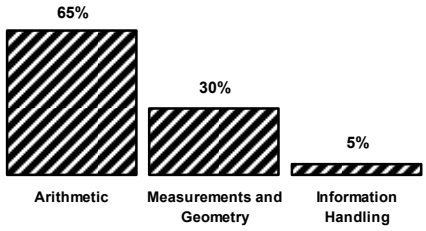
UNITWISE WEIGHTAGES – GRADE II

Unit	Title	Weightage
1.	Numbers	30 %
2.	Number Operations	35 %
3.	Measurement of Length, Mass and Capacity	10 %
4.	Time	10 %
5.	Geometry	15 %
	TOTAL	100 %



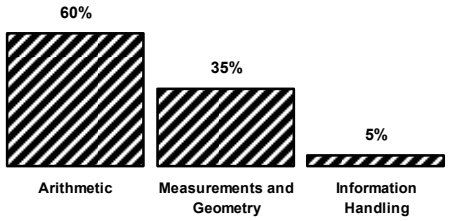
UNITWISE WEIGHTAGES – GRADE III

Unit	Title	Weightage
1.	Numbers	15 %
2.	Number Operations	20 %
3.	Fractions	20 %
4.	Measurement of Length, Mass and Capacity	15 %
5.	Time	10 %
6.	Geometry	15 %
7.	Data Representation	5 %
	TOTAL	100 %



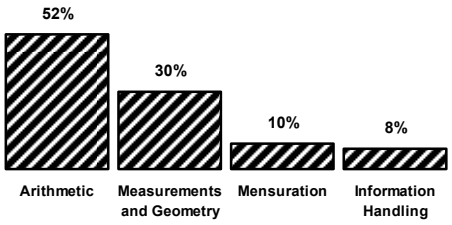
UNITWISE WEIGHTAGES – GRADE IV

Unit	Title	Weightage
1.	Numbers and Arithmetic Operations	15 %
2.	Factors and Multiples	15 %
3.	Fractions	15 %
4.	Decimals and Fractions	15 %
5.	Measurements	15 %
6.	Geometry	20 %
7.	Information Handling	5 %
	TOTAL	100 %



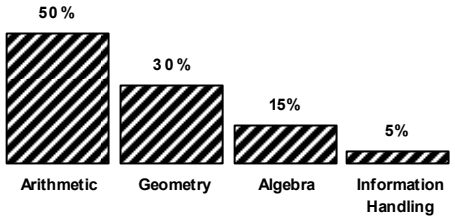
UNITWISE WEIGHTAGES – GRADE V

Unit	Title	Weightage
1.	Numbers and Arithmetic Operations	10 %
2.	HCF and LCM	10 %
3.	Fractions	10 %
4.	Decimals and Percentages	7 %
5.	Distance, Time and Temperature	10 %
6.	Unitary Method	15 %
7.	Geometry	20 %
8.	Perimeter and Area	10 %
9.	Information Handling	8 %
	TOTAL	100 %



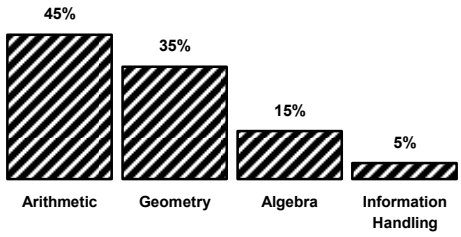
UNITWISE WEIGHTAGES – GRADE VI

Unit	Title	Weightage
1.	Sets	5 %
2.	Whole Numbers	5 %
3.	Factors and Multiples	20 %
4.	Integers	5 %
5.	Simplifications	5 %
6.	Ratio and Proportion	5 %
7.	Financial Arithmetic	5 %
8.	Introduction to Algebra	7 %
9.	Linear Equations	8 %
10.	Geometry	15 %
11.	Perimeter and Area	7 %
12.	Three Dimensional Solids	8 %
13.	Information Handling	5 %
	TOTAL	100 %



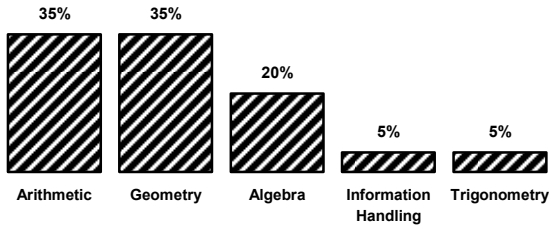
UNITWISE WEIGHTAGES – GRADE VII

Unit	Title	Weightage
1.	Sets	7 %
2.	Rational Numbers	7 %
3.	Decimals	7 %
4.	Exponents	7 %
5.	Square Root of Positive Number	6 %
6.	Direct and Inverse Variation	6 %
7.	Financial Arithmetic	5 %
8.	Algebraic Expressions	10 %
9.	Linear Equations	5 %
10.	Fundamentals of Geometry	12 %
11.	Practical Geometry	15 %
12.	Circumference, Area and Volume	8 %
13.	Information Handling	5 %
	TOTAL	100 %



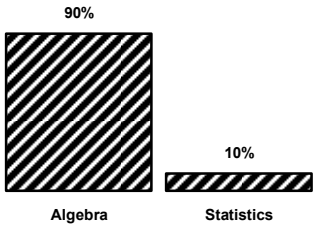
UNITWISE WEIGHTAGES – GRADE VIII

Unit	Title	Weightage
1.	Operations on Sets	7 %
2.	Real Numbers	12 %
3.	Number Systems	8 %
4.	Financial Arithmetic	8 %
5.	Polynomials	5 %
6.	Factorization, Simultaneous Equations	15 %
7.	Fundamentals of Geometry	7 %
8.	Practical Geometry	12 %
9.	Areas and Volumes	6 %
10.	Demonstrative Geometry	10 %
11.	Introduction to Trigonometry	5 %
12.	Information Handling	5 %
	TOTAL	100 %



UNITWISE WEIGHTAGES – GRADES IX-X

Unit	Title	Weightage
1.	Matrices and Determinants	10 %
2.	Real and Complex Numbers	6 %
3.	Logarithm	5 %
4.	Algebraic Expressions and Algebraic Formulas	8 %
5.	Factorization	8 %
6.	Algebraic Manipulation	5 %
7.	Linear Equations and Inequalities	5 %
8.	Quadratic equations	7 %
9.	Theory of Quadratic equations	13%
10.	Variations	6 %
11.	Partial fractions	5 %
12.	Sets and Functions	12 %
13.	Basic Statistics	10 %
	TOTAL	100 %



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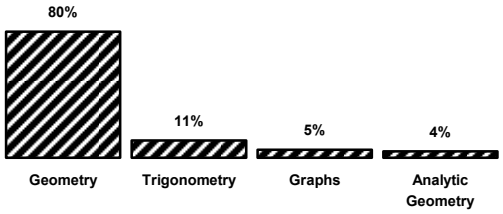
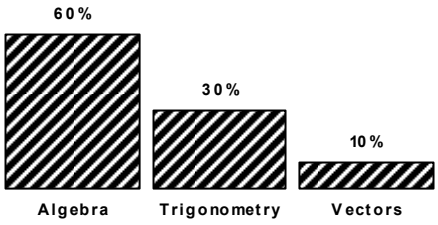
Unit	Title	Weightage
14.	Linear Graphs and their Applications	5 %
15.	Introduction to Coordinate Geometry	4 %
16.	Introduction to Trigonometry	11 %
17.	Congruent Triangles	6 %
18.	Parallelograms and Triangles	6 %
19.	Line Bisectors and Angle Bisectors	6 %
20.	Sides and Angles of a Triangle	5 %
21.	Ratio and Proportion	6 %
22.	Pythagoras Theorem	8 %
23.	Theorems related to Area	5 %
24.	Projection of a Side of a Triangle	6 %
25.	Chords of a Circle	5 %
26.	Tangent to a Circle	6 %
27.	Chords and Arcs	5 %
28.	Angle in a Segment of a Circle	6 %
29.	Practical Geometry – Triangles	3 %
30.	Practical Geometry – Circles	7 %
	TOTAL	100 %



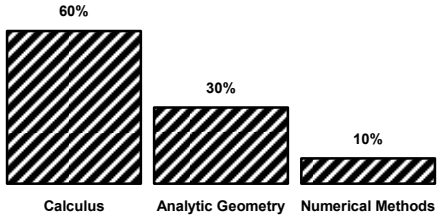
UNITWISE WEIGHTAGES – GRADE XI

Unit	Title	Weightage
1.	Complex Numbers	6 %
2.	Matrices and Determinants	15 %
3.	Vectors	10 %
4.	Sequences and Series	6 %
5.	Miscellaneous Series	3 %
6.	Permutation, Combination and Probability	8 %
7.	Mathematical Induction and Binomial Theorem	8 %
8.	Functions and Graphs	9 %
9.	Linear Programming	5 %
10.	Trigonometric Identities of Sum and Differences of Angles	8 %
11.	Application of Trigonometry	7 %
12.	Graphs of Trigonometric and Inverse Trigonometric Functions and Solution of Trigonometric Equations	15 %
	TOTAL	100 %



UNITWISE WEIGHTAGES – GRADE XII

Unit	Title	Weightage
1.	Introduction to Symbolic Package: MAPLE	0 % (assumed initially)
2.	Functions and Limits	11 %
3.	Differentiation	11 %
4.	Higher Order Derivatives and applications	4 %
5.	Differentiation of Vector Functions	4 %
6.	Integration	18 %
7.	Plane Analytic Geometry– Straight Line	13 %
8.	Conics-I	5 %
9.	Conics-II	12 %
10.	Differential Equations	8 %
11.	Partial Differentiation	4 %
12.	Introduction to Numerical Methods	10 %
	TOTAL	100 %



Teaching and Learning Resources

Introduction

Government prescribed textbook is the only teaching and learning tool used in most of the schools. Though many other resources are also available, accessible and affordable teachers rarely use them to support the learning. In addition to the textbook, the teaching and learning resources include teacher's manual, workbook and electronic resources.

Part I The Textbook

There are many important entities involved to revamp the entire education system. The school has to play its own role, parents have to contribute their share and teachers have to assume a significant place in fostering education. Print materials, particularly the textbooks, have to play a key role towards providing quality education at all levels. Although there are many stakeholders that contribute towards the overall learning of the child yet the importance of textbook as a reservoir of information/ knowledge cannot be ignored.

Textbook writers have a vital role to play in penetrating the young minds through their writing. A textbook

- whose content as well as presentation is thoughtfully planned,
- which is written by qualified and competent subject expert(s), and
- which is attractive and engaging,

must stimulate the interest of teacher and the taught.

Guidelines for Textbook Authors

Textbooks aimed at lower level tend to include more learning features than those at higher level. However in textbook writing generally the following aspects may be taken into consideration.

- The textbook should be in line with the objectives of National Curriculum.
- The author should continuously focus on standards and benchmarks.
- The author should bring himself to the mental level of students he is writing for.
- The span of the textbook should be fairly reasonable.
- The material should not be cramped. To make it more digestible, it may be chunked into smaller parts with headings.
- The textbook is expected to provide accurate and up-to-date information.
- The text material should be arranged in a logical manner; simple to complex, familiar to unfamiliar and concrete to abstract.
- The text material must be free from ambiguities and errors (both mathematical and typographical).
- The content provided in the textbook should not develop wrong concepts.
- The text should be clear and concise. It should not give any other meaning than the one intended.
- Special attention should be paid to geometrical portions. Every table, line drawing and graph should be labeled appropriately.
- Footnotes and side notes may be inserted wherever necessary.

Textbook Style and Structure

To make a textbook an effective teaching and learning tool its style and structure is given due importance. The material needs to be structured in a coherent and logical way, and that writing style should be reader friendly.

Unit Opening	
Unit Outline	Include list of headings.
Student Learning Outcomes (SLOs)	One SLO for each heading may be included. If they are numerous then a reasonable number is acceptable.
Real Life Relevance	Illustrate the real life relevance of the unit, if possible.
Short Introduction	Explain what this unit covers and why.

Unit Body	
Key Terms	Use italics for emphasis and bold for key terms. Define key terms when first introduced and collate them with their definitions for the glossary.
Running Glossary	Key terms and definitions may be pulled out from the main body of text so that students spot them easily in the unit body (e.g. in the margins).
Feature Boxes	Regular feature boxes may include various contents such as a mathematical formula, a working rule or a statement of theorem.
Illustrative Examples	Include illustrative examples to develop conceptual understanding of the topic.
Problem Sets	Special attention should be paid on preparation of Problem Sets. Correlate Mathematics with real life situations and include sufficient exercises on real life problems almost in every problem set, if appropriate. The questions on the application of Mathematics in other fields of study are also very useful.
Learning Review Points	Include bulleted questions for students to check their understanding at regular intervals. Possible labels include 'self-test point' or 'checkpoint'.
Tips or Hints	Separated from the main body of text, they allow the author to speak directly to the student, offering useful advice or flagging important points.
Visuals	Tables, graphs, line drawings and lists may be used to break up the text.

Unit Ending	
Problem Set (Review)	Include multiple-choice questions, interpretive exercises and fill-in items. Students may also be asked to label diagrams or write a one word answer to short question.
Summary	Include a review of the main concepts. This can relate to the SLOs by covering each in turn (bullet points work well). The summary should not include any new information.

End of Textbook	
Glossary	Include only the key terms in the glossary.
Answers to Problems	Include answers to the problem sets unit wise.
Appendices	Include extra information the student needs such as list of mathematical formulas, log tables and relevant websites.
Bibliography	Include bibliography and list of books for suggested reading where appropriate.
Index	Include index for the key terms used in the book.

Additional Instructions for Authors – Grades IX-X

A few additional but important instructions for authors are given below.

- There will be two books published for grades IX-X.
- The book titled **Mathematics-I (Algebra)** will cover the units 1-13 while the units 14-30 will be swathed in the book titled **Mathematics-II (Geometry)**.
- Before unit 17 a page titled 'Descriptive Geometry' should be inserted as a separator.
- Both the books should clearly highlight the following instructions for students and teachers.

  (a) Units 1-7, 14-15, 17-23 and 29 will be taught at grade level IX.
  (b) Units 8-13, 16, 24-28 and 30 will be taught at grade level X.
- In Mathematics-II units 14-30 may be relabeled to be read as units 1-17.

Part II The Teacher's Manual

Ideally the teacher's manual should come with the textbook. The manual is aimed at informing teachers how the textbook is written and how best to use it to facilitate student learning. It can be seen as a means of helping teachers develop professionally. It provides detailed explanation of key concepts and the way to teach a particular topic. Its basic features are as below.

The teacher's manual should

- be easy to understand and use.
- help teachers teach text and extend activities.
- give sequenced instructions for each activity.
- include teaching learning resources.
- establish a question bank (having questions different from text) and suggest interactive quizzes corresponding to each unit.
- involve various up-to-date and relevant teaching strategies and rationale for suggested teaching.
- explain how to implement each teaching strategy.
- identify constraints and strengths of each strategy or activity.

- identify resources needed for teaching strategies and extension of activities.
- expand and develop teachers repertoire of knowledge and skills.
- identify assessment strategies.

Part III The Workbook

Workbooks contain writing activities and exercises that are related to each unit in the textbook. Workbook exercises help to develop students' conceptual understanding of the topics dealt with in the text. They assist students in developing skills by applying knowledge to new situations. A workbook has the following basic features.

A workbook should

- be easy for students to understand and follow.
- involve clear and explicit instructions.
- be stimulating, challenging and innovative.
- correspond to knowledge and skill developed in the textbook.
- consist of many exercises and activities for each unit, topic and subtopic.
- be non-repetitive in style and structure.
- avoid using too many activities for one topic or skill.
- include exercises and activities which are different from those in textbook or teacher's manual.
- suggest accessible and affordable materials/resources for the proposed activities.

There are a number of teaching and learning materials available for effective teaching. Following are a few examples of easily accessible and affordable materials which can be suggested for the activities corresponding to basic concepts of Mathematics.

Materials	Purpose
Mirror	symmetry
Cards, coins, dice and marbles	probability
Food, paper or something that can be divided into fractions	fraction
Ruler, set squares, protractor, string, thermometer and weights	measurements
3-dimensional objects; ball, cube, cylinder, prism and pyramid	volume and surface area
Graphs, pie charts and multiplication tables	visual aids

Part IV The Web-based Resources

The use of world wide web (www) is growing very fast to access an immense volume of rapidly evolving information. It is acting as a driving force since its ease of use makes the internet trivially accessible to the students even with a little knowledge of computer. Through web-based links as mentioned below the teachers, parents and students will be able to

- access various sites of Mathematics around the world,
- view three-dimensional figures, graphics, lesson plans, activities and various books of interest.

Title of Website	Universal Resource Locator (URL)
A+Math	http://www.aplusmath.com/
AAA Math	http://www.aaamath.com/
Academic Info-Mathematics	http://www.academicinfo.net/math.html
Algebra Buster	http://www.algebra-online.com/
Algebra Helper	http://www.algebrahelp.com/index.jsp
Class Zone	http://www.classzone.com/math_middle.cfm
Click on Bricks	http://kathyschrock.net/clickonbricks/index2.htm
Cool Math	http://www.coolmath.com/
Discovery School (Mathematics)	http://school.discovery.com/lessonplans/math.html
Frank Potter's Science Gems-Mathematics	http://www.sciencegems.com/math.html
Funbrain	http://www.funbrain.com/numbers.html
Geometry	http://www.mathleague.com/help/geometry/geometry.htm
Internet Mathematics Library	http://www.mathforum.org/library
MAPLE	http://www.maplesoft.com
Math Archives	http://www.archives.math.utk.edu/
Math Glossary	http://www.harcourtschool.com/glossary/math_advantage
Math Goodies	http://www.mathgoodies.com
Math World	http://www.mathworld.wolfram.com
Math2	http://www.math2.org/
MATHEMATICA	http://www.wolfram.com/products/mathematica/index.html
Mathematical Interactivities	http://mathematics.hellam.net/
MathStories	http://www.mathstroies.com
Mega Mathematics	http://www.c3.lanl.gov/mega-math/
Purplemath	http://www.purplemath.com/internet.htm
S.O.S. Mathematics	http://www.sosmath.com
Superkids Educational Software Review	http://www.superkids.com/aweb/tools/math/index.shtml
Teaching madeEasier	http://www.teachingmadeasier.com/math.html
The MathWorks (MATLAB)	http://www.mathworks.com
Webmath	http://www.webmath.com/

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